On the Reversibility of Entanglement Manipulation

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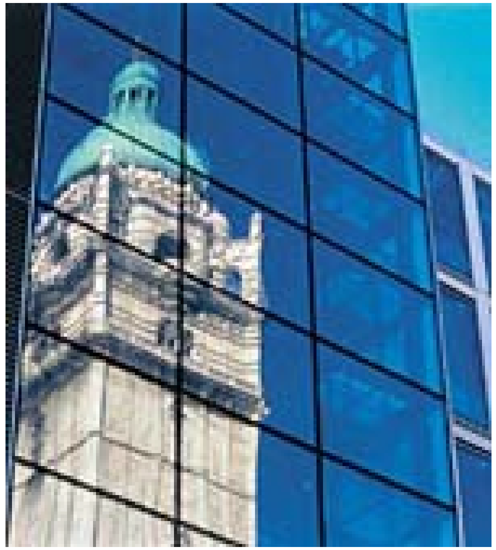

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One main theoretical challenge in Quantum Information Theory:

Characterization of quantum entanglement

Three basic questions:

- Is a given state entangled or not?
- If it is entangled, how much entanglement does it contain?
- When and how can we locally transform a given entangled state into another?

• Qualify • Quantify • Manipulate	}	Questions addressed in EQUIP

Disentangled states: $\rho_{sep} = \sum_i p_i \rho_A^i \otimes \rho_B^i$ = separable state

(Convex combination of product states)

Can be created by local operations and classical communication!

Entangled states: Can't be written in the form $\rho_{ent} \neq \sum_i p_i \rho_A^i \otimes \rho_B^i$

Creation requires coherent transfer of quantum particles!

LOCC cannot increase entanglement!

Given ρ, is it separable or not? Efficiently solvable for qubits

Example:

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \Leftrightarrow \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \rightarrow \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

β β

ρ $\rho^{\bullet}$

$\text{Eigenvalues}: \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\}$

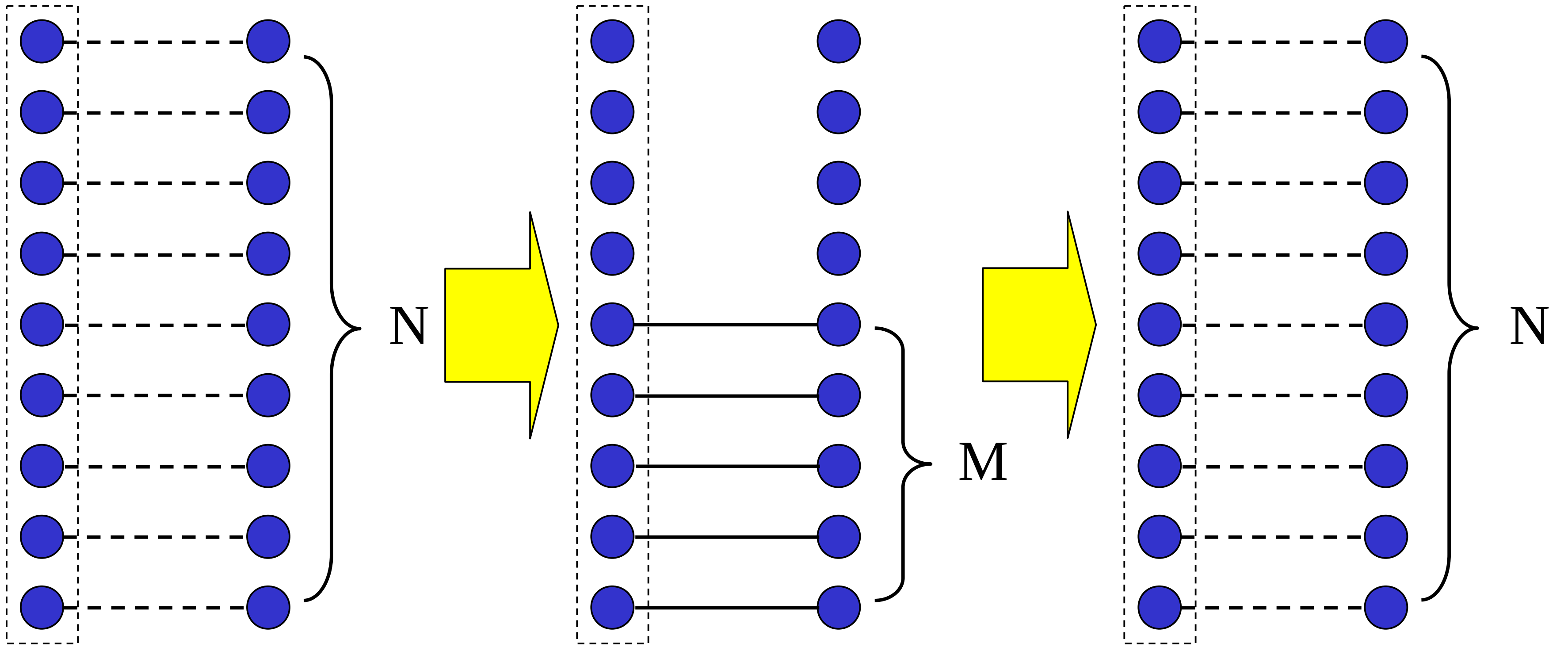
The state is not separable if one of the eigenvalues of $\rho^{\bullet}$ is positive.

This motivates to use $\log_2 tr|\rho^\Gamma|$ as a measure of entanglement
(Werner 1997, but operational meaning of $\log_2 tr|\rho^\Gamma|$ unclear)

Distillation: By LOCC obtain a sub-ensemble of more entangled states

Necessary condition: Partial transpose is non-positive

Distillation in the asymptotic limit of many identical pure states à Reversibility



$\alpha|00\rangle + \beta|11\rangle$ $\qquad$ $\dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$ $\qquad$ $\alpha|00\rangle + \beta|11\rangle$

$$\text{Efficiency} = \frac{M}{N} \xrightarrow[N\to\infty]{} -\alpha^2 \log \alpha^2 - \beta^2 \log \beta^2 = S(\rho_A)$$

Entropy of entanglement

Procedure: Alice determines the number of particles in state |1>

(Energy measurement on all her particles)

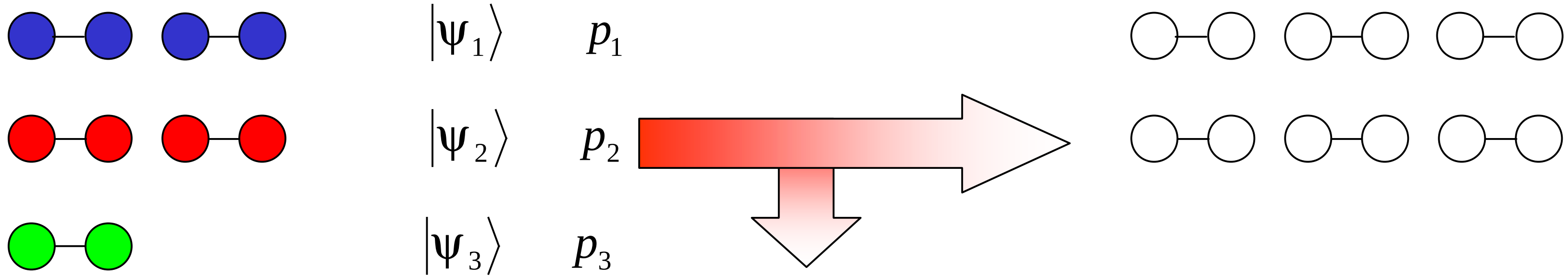
Number of possible outcomes for n pairs = n+1

Entropy $H \approx \log(n+1)$ Entropy per pair $\approx \frac{\log n}{n} \xrightarrow[n \to \infty]{} 0$

Information/ebit obtained in the measurement vanishes asymptotically

Expect that procedure becomes asymptotically reversible!

Mixed states emerge due to loss of classical information:



Labels are erased $kT\,H(\{p_i\})$ of heat generated per system

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

This lost information has to be retrieved via local measurements!

Finite amount of information per system and per ebit has to be obtained

Expectation: Irreversibility as measurements will destroy some of the entanglement

Entanglement measures

Any quantity that does not increase under LOCC and one pure states equals the entropy of entanglement $S(\rho_A)$

Entanglement cost:

How many maximally entangled states are required to prepare the mixed state r by LOCC?

Other entanglement measures lie in between the two, e.g.

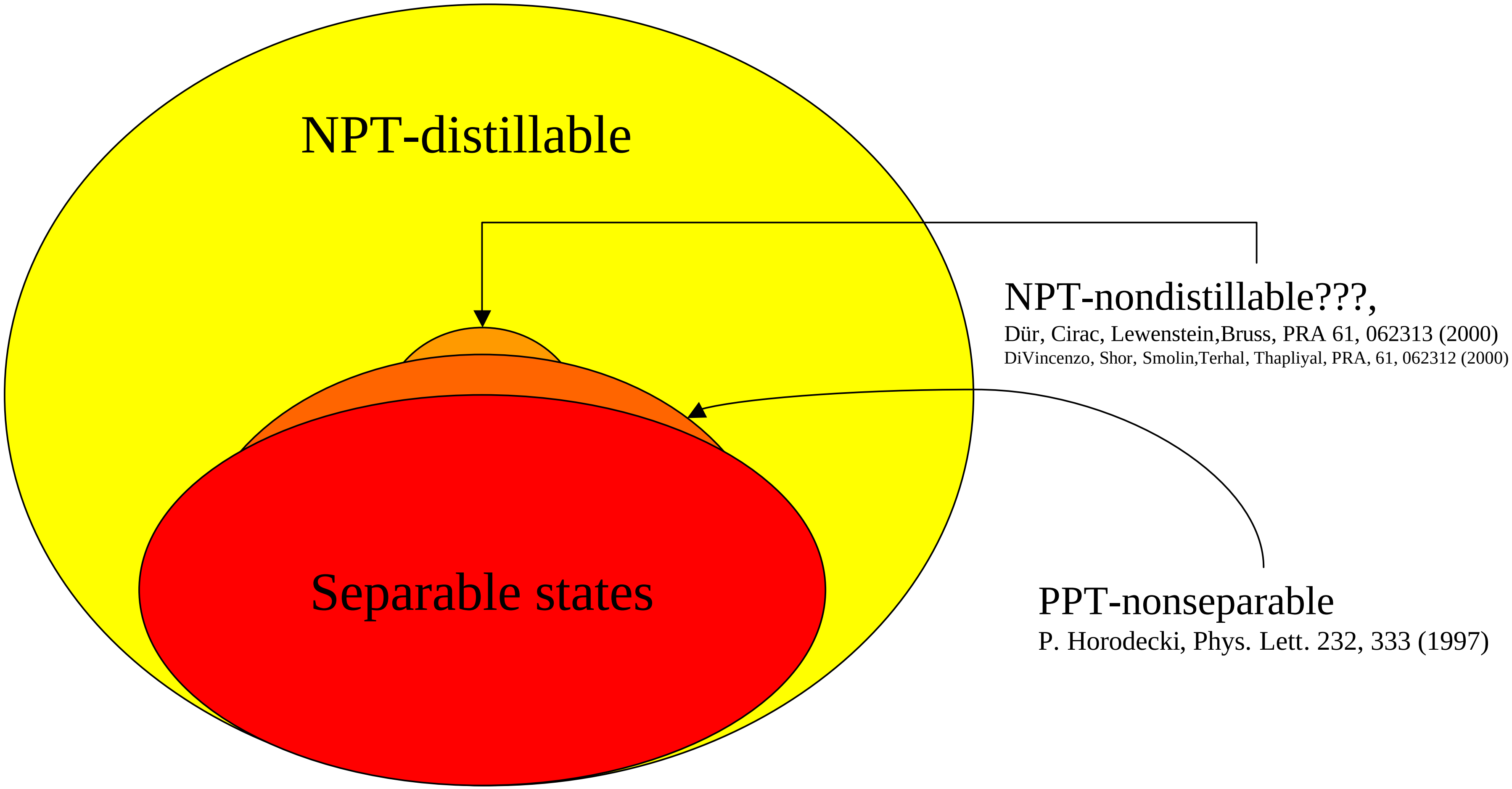
Relative entropy of entanglement, measures the distinguishability between an entangled state and disentangled states.

Entanglement of distillation:

How many maximally entanglement states can one obtain by LOCC from a mixed entangled state r .

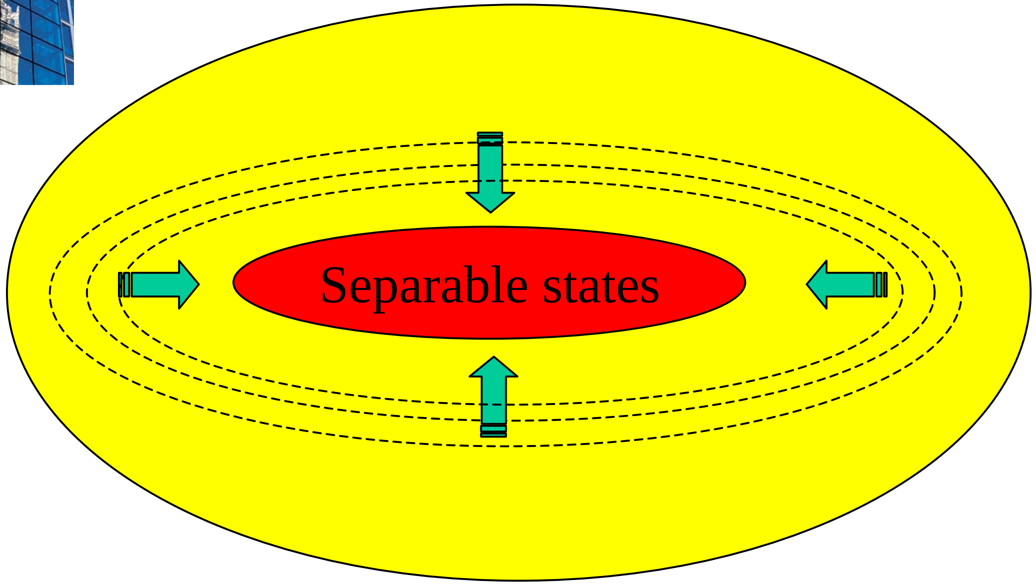
Reversibility of LOCC manipulation $\Leftrightarrow E_C = E_D$

We have $E_C \geq E_D$ but are they equal or not?

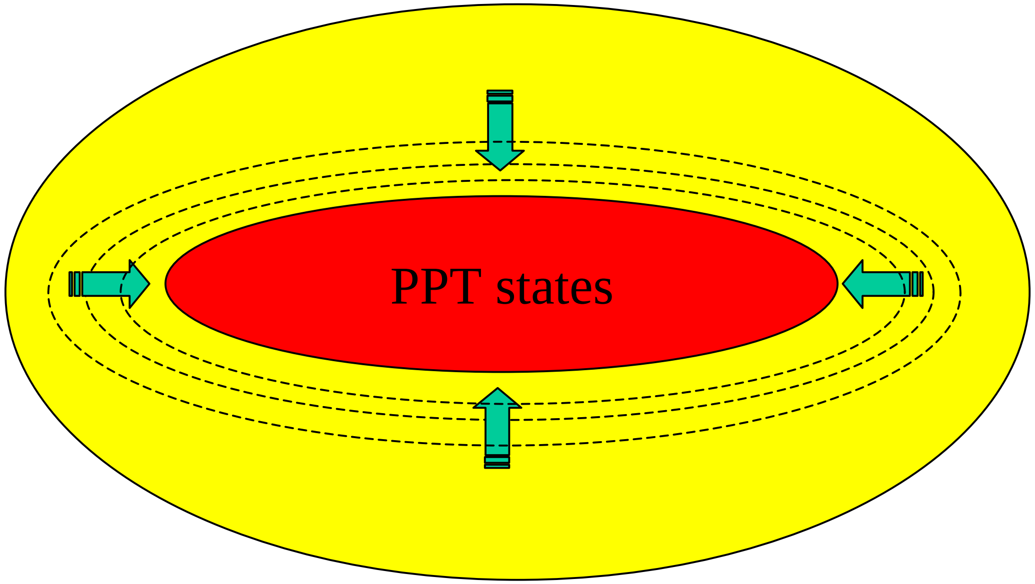


PPT-nonseparable states can be shown to have $E_D = 0 \leq E_C$

LOCC Entanglement manipulation on mixed states is irreversible!



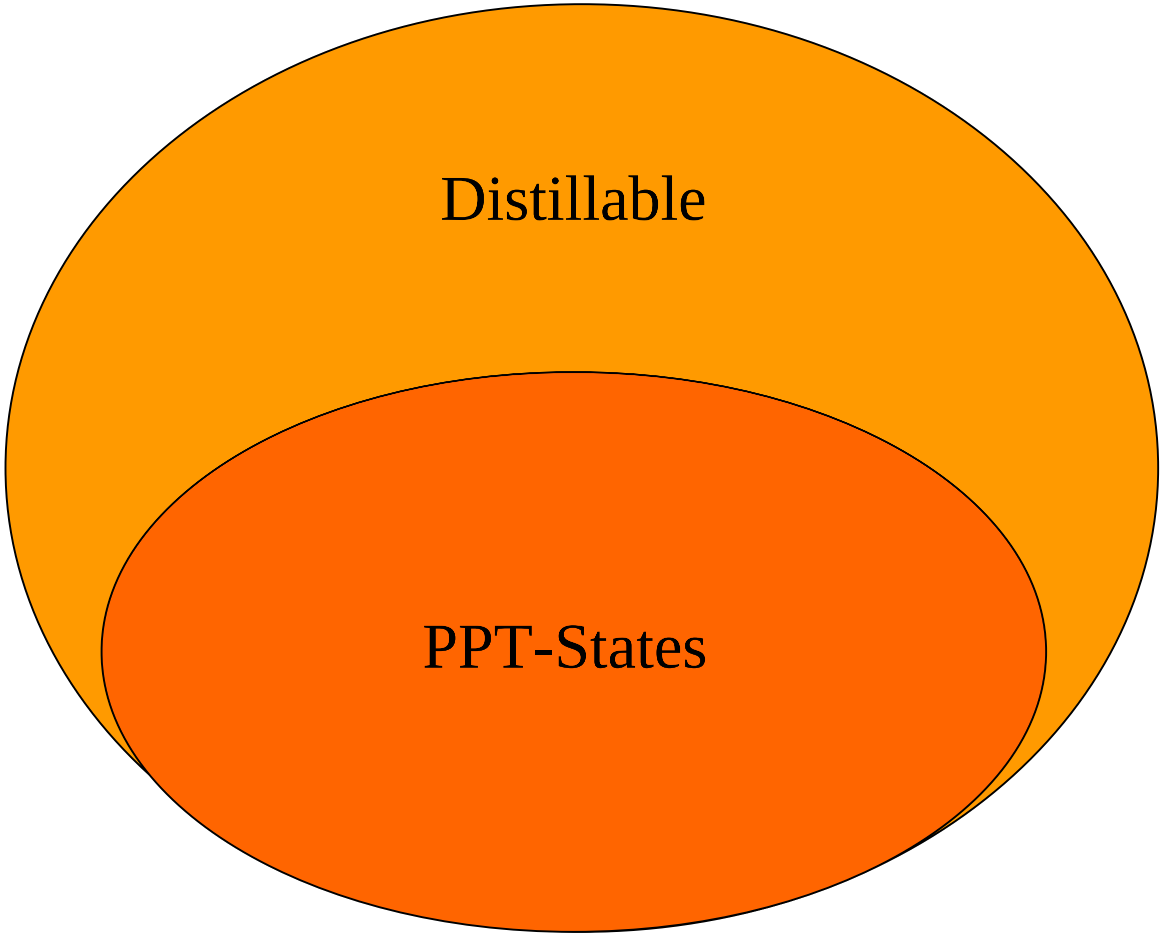
LOCC operations leave set of separable state invariant



PPT operations leave the set of PPT states invariant

Advantage I: PPT operations can be characterized efficiently

Advantage II: Structure of state space under PPT-operations is simpler



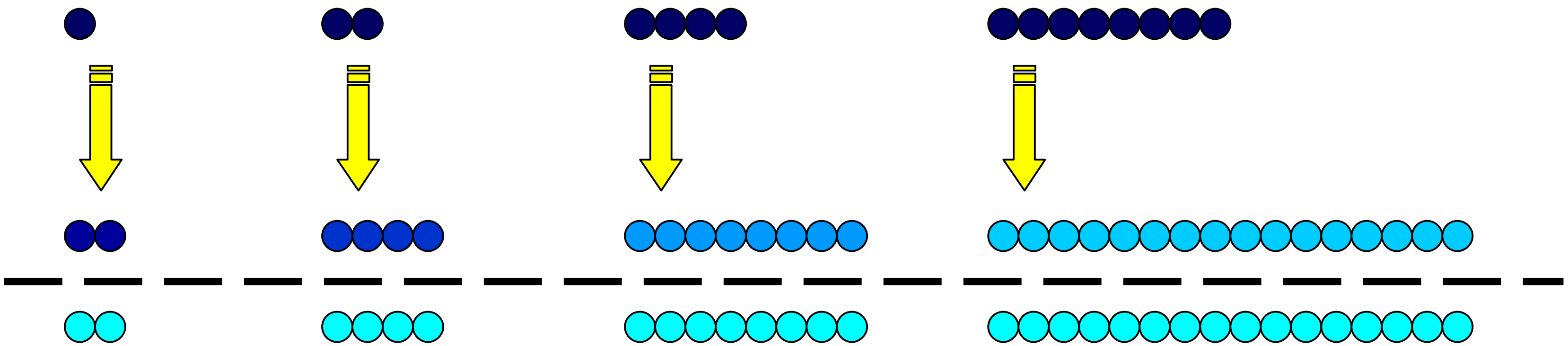
Eggeling, Vollbrecht, Werner, Wolf, PRL 87, 257902 (2001).

Reconsider the entanglement cost now under PPT-operations:

Version 1: Asymptotically exact preparation r

$$C_{ppt}(\rho) = \inf\{r : \lim_{n\to\infty} \inf_{\Psi} tr\,|\,\rho^{\otimes n} - \Psi(\Phi(2^{rn}))\,| = 0\}$$

Where $\Phi(n) = \frac{1}{n}(\sum_{k=1}^{n} |kk\rangle)(\sum_{k=1}^{n} \langle kk|)$ and $\Psi = \text{PPT - operation}$



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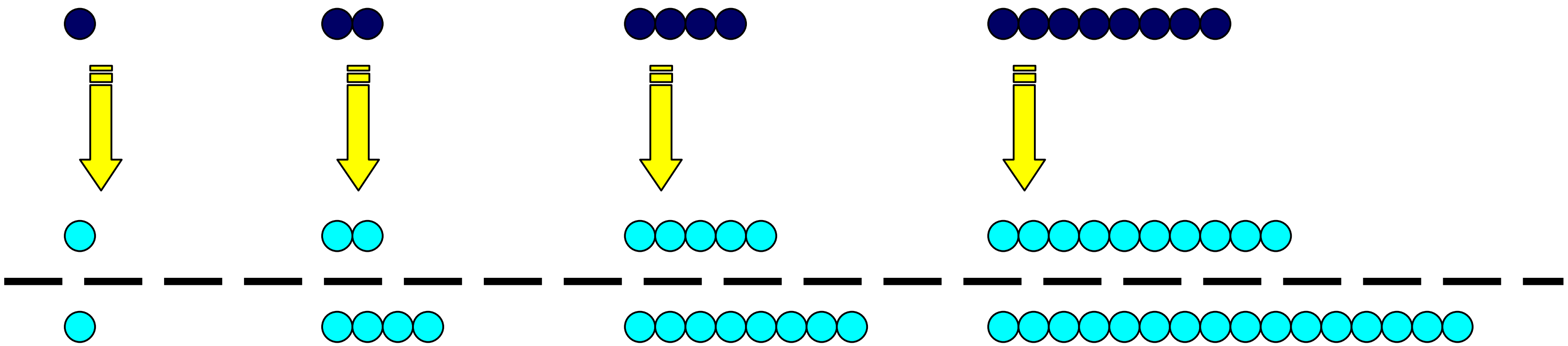
Reconsider the entanglement cost now under PPT-operations:

Version 2: Exact preparation of r for any number of copies

$$E_{ppt}(\rho) = \lim_{n\to\infty} \inf\{r_n : \inf_{\Psi} tr\,|\,\rho^{\otimes n} - \Psi(\Phi(2^{r_n n}))\,| = 0\}$$

Where $\Phi(n) = \frac{1}{n}(\sum_{k=1}^{n} |kk\rangle)(\sum_{k=1}^{n} \langle kk|)$ and $\Psi = \mathrm{PPT\text{-}operation}$



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Theorem: The PPT-entanglement cost $E_{ppt}(\rho)$ for the exact preparation of the state r satisfies

$$\log_2 tr\,|\rho^\Gamma| \leq E_{ppt}(\rho) \leq \log_2 Z(\rho)$$

where

$$Z(\rho) = \mathrm{tr}\,|\rho^\Gamma| + \dim(\rho)\max(0, -\lambda_{\min}(|\rho^\Gamma|^\Gamma))$$

Examples: $|\rho^\Gamma|^\Gamma \geq 0$ is satisfied

- in finite dimensions: Pure states, Werner states, …
- in infinite dimensions (CV-systems): All Gaussian states, …

For all these states we know that $E_{ppt}(\rho) = \log_2 tr\,|\rho^\Gamma|$!

e Operational interpretation of the logarithmic-negativity!

Lemma: The PPT-entanglement cost C_{ppt} for the anti-symmetric Werner state $\rho = \sigma_a$ is given by $\log_2 tr|\rho^\Gamma|$ and coincides with its distillable entanglement.

Proof: $C_{ppt}(\rho) \leq E_{ppt}(\rho) = \log_2 tr|\rho^\Gamma|$

For this state one can also compute the distillable entanglement under PPT-operations.

$$E_{D,ppt}(\rho) = \log_2 tr|\rho^\Gamma|$$

$$\Rightarrow \quad E_{D,ppt} \leq C_{ppt}(\rho) \leq E_{ppt}(\rho) = \log_2 tr|\rho^\Gamma| = E_{D,ppt}$$

All quantities coincide, PPT-entanglement manipulation for this mixed state is reversible!

Numerial evidence suggests that this statement is true for all Werner states!

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Conjecture: Under PPT-operations entanglement manipulation becomes reversible and, as a consequence, there is a unique entanglement measure, which is the asymptotic relative entropy of entanglement.

Conclusions:

- LOCC entanglement manipulation is irreversible in general
- Reversibility may be retrieved under PPT-operations
- In that case there would be a unique entanglement measure which coincides with the relative entropy of entanglement

Outlook:

- Decide conjecture of reversibility under PPT-operations
- Understand what is so special about PPT-operations