Capacities of Two-Qubit Unitary Operations

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Abstract: Two-qubit operations may be characterised by their capacities for communication, both with and without free entanglement, and their capacity for creating entanglement. We establish a set of inequalities that give an ordering to the capacities of two-qubit operations. In addition we present numerical evidence of equalities between capacities in some cases, and strict inequalities in other cases.

Quantum information processing relies on applications of single-qubit unitary transformations and nonlocal two-qubit unitary transformations, where the term "nonlocal" refers to joint operations on separate Hilbert spaces. These nonlocal operations are powerful tools in quantum information theory and can be characterised by the entanglement they can generate between the two Hilbert spaces as well as by the communication capacity they can deliver. These two seemingly disparate measures of gate strengths are useful for quite different applications of two-qubit nonlocal gates. However, we establish that these capacities are closely related.

The capacity for creating entanglement can be treated in a relatively straightforward way. One may consider the entanglement that may be created from initially unentangled states [1], or the entanglement increase when the initial state is entangled [2]. The communication capacity is a more difficult quantity to consider, because most operations can not be used to perform communication without error. Initial research focused on the communication that can be performed using simple gates, such as the CNOT and SWAP gates [3, 4].

One way of defining communication capacities for general operations is to consider the communication per operation in the limit that the operation is performed a large number of times [5]. When communication is performed in this way, the probability of error may be made arbitrarily small. An alternative way of defining communication capacities is by considering the communication that may be performed using an ensemble [5, 6].

It is known that there is equality between many of these capacities in the case of the CNOT or SWAP operations [3, 4]. We show that, although it is not known if these equalities hold for general operations, it is possible to establish an ordering. In particular we show that the communication that it is possible to perform without entanglement assistance is at least as great as half the communication it is possible to perform with entanglement assistance. This inequality holds both for communication in a single direction and for bidirectional communication.

We establish that the capacity based on the maximum increase in the bidirectional communication that may be performed using an ensemble is at least as great as twice the capacity for entanglement. Together with the inequality between the unassisted communication capacity and entanglement capability shown in Ref. [5], these results give an ordering to the various capacities.

We also present numerical results on the communication capacity in a single direction. The entanglement-assisted capacity can be calculated via the maximum increase in the communication that may be performed using an ensemble, given by the Holevo information [5]. Our results demonstrate a strict inequality between this capacity and the capacity for creating entanglement. In addition, we present numerical results for the maximum Holevo information that an operation may create from an initial ensemble with zero Holevo information. These results show that in many cases this capacity is equal to the maximum entanglement that may be created from an initially unentangled state, indicating that these capacities are closely related.

References

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