Entanglement Dynamics in 1D Quantum Cellular Automata

Gavin K. Brennen and Jamie E. Williams
National Institute of Standards and Technology, Gaithersburg, MD 20899-8423, USA

Several proposed schemes for the physical realization of a quantum computer consist of qubits arranged in a cellular array. In the quantum circuit model of quantum computation, an often complex series of two-qubit gate operations is required between arbitrarily distant pairs of lattice qubits. An alternative model of quantum computation based on quantum cellular automata (QCA) requires only homogeneous local interactions that can be implemented in parallel. This would be a huge simplification in an actual experiment. We find some minimal physical requirements for the construction of unitary QCA in a 1 dimensional Ising spin chain and produce optimal pulse sequences for information transport and entanglement distribution. We also introduce the theory of non-unitary QCA and explain how they can be constructed using an ancillary lattice of controllable single qubit environments. We show, by example, that the mixing of a non-unitary rule with a unitary rule can generate entanglement across a spin chain where there is none for purely unitary evolution. This is an example of environment assisted entanglement generation.

A primary motivation for studying QCA is to use low computational depth circuits applied in uniform across the system to produce complex quantum dynamics. Most previous work on QCA has focused on mapping such systems to the circuit model [1, 2]. Additionally, there have been investigations of quantum lattice gas automata (QLG) for simulations of the Dirac equation in 1D and for topological computation [3, 4]. Recently there was an experimental realization in liquid state NMR of a QLG algorithm to solve the classical 1D diffusion equation [5]. Our focus is the interplay between quantum and classical computation, i.e. between quantum and classical correlations in a discrete dynamical system. Characterizing multi-particle entanglement is a field of active research both for its potential use in quantum information processing and for understanding non-locality in nature. QCA can offer a unique approach to study the raw computational effort needed to generate such entanglement.

From an experimental standpoint, a QCA has a significant advantage over a traditional quantum computer because individual qubits in the lattice do not need to be separately addressed and the system is modular and reconfigurable. In an implementation, applying uniform fields over the entire system helps to eliminate error resulting from cross talk on neighboring qubits due to imperfectly aligned control fields. Some specific physical systems have been proposed as candidates for QCA including quantum dot arrays [6] and endohedral fullerenes [7]. In presenting the QCA formalism, we provide specific examples of possible experimental implementations in order to emphasize the relevance of the QCA approach to present day technologies.

[1] J. Watrous, Proceedings of the 36th IEEE Symposium on Foundations of Computer Science p. 528 (1995).
[2] S. C. Benjamin, Phys. Rev. A **61**, 020301 (2000).
[3] D. A. Meyer, J. Phys. A **34**, 6986 (2001).
[4] D. A. Meyer, J. Stat. Phys. **85**, 551 (1996).
[5] M. A. Pravia, Z. Chen, D. G. Cory, and J. Yepez, quant-ph/0303183 (2003).
[6] G. Toth and C. S. Lent, Phys. Rev. A **63**, 052315 (2001).
[7] J. Twamley, quant-ph/0210202 (2002).