Quantum trajectories and quantum information: teleportation in a noisy environment

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Simulating a quantum protocol amounts to solve a master equation, since quantum processors should be entangled with their environments in real situations, and therefore their states are described by density matrices. Solving the master equation for a state of several qubits can prove to be a prohibitive task in terms of memory cost. Instead of doing so, quantum trajectories allow for storing only a stochastically evolving state vector. By averaging over many runs we get the same probabilities (within the statistical error) as the ones obtained by solving for the density matrix directly. We find that a reasonable number of trajectories ($\mathcal{O}(100)$) is needed in order to find a satisfactory convergence.

As it has been pointed out by Brun [1], outside the quantum optics and quantum foundations fields, the theory of quantum trajectories is almost unknown. They were intended to model continously monitored open systems [2, 3, 4], in fact, systems that interact with their environments. But there are a few cases where they have been used or suggested for calculations in quantum information [5, 6]. One of the main objectives of this talk is to contribute to bridge this gap.

We apply this formalism to study the behaviour of the quantum teleportation protocol [7] through a large chain of qubits. Teleportation is one of the basic methods of quantum communication, thus being one of the most important areas in quantum information theory. Also it has been proven that is a primitive for universal quantum computation. A noiseless quantum channel is required in order to share maximally entangled particles, despite the available ones are typically noisy. In our model we perform the delivering of one of the bits of the pair using swap gates along a qubit chain. This chain is an open system that interacts with its environment through the amplitude damping channel. We are able to study the fidelity of teleportation defined as the overlap between the reduced density matrix of Bob's qubit and the original unknown state. We do so for different lengths of the chain making a comparison with the direct solution of the Lindblad master equation. Moreover, we explore different coupling strengths with the environment. We have been able to obtain exact results (within the statistical error) for an open system of up to 25 qubits in presence of an amplitude damping environment, using a cluster of PC-type processors.

Teleportation has become an active field not only for theoretical developments but also for actual and proposed experiments. Then, it is possible that in the near future teleportation could be implemented at large scale in any of the proposed solid state devices for quantum computation. In any case, for the design and future construction of quantum hardware, previous simulations become essential. Given the ability of a single quantum trajectory to provide a very good idea of an individual experimental run, a natural objective of the proposed talk is to present a convenient framework to model experiments.

Our example opens many possibilities for future studies. Theoretical predictions for fidelity and entanglement measures behaviour with respect to system size and different kinds of environment can now be explored with the help of numerical simulations. One of our present lines of research is guided towards modelling different noise channels and introducing a realistic internal Hamiltonian. On the other hand, different protocols can be easily implemented, such that the scope of the present approach can be extended to study their stability.

References

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