LOCC entanglement convertibility in infinite dimensional systems

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Abstract

Entanglement is regarded as the key resource which allows many quantum information processing schemes to out-perform their classical counterparts. The convertibility of two different entangled states (either for single copy cases or multi-copy cases) under LOCC (local operations and classical communications) is important for the qualitative and quantitative understanding of entanglement. For finite dimensional bipartite systems, we now have a better understanding of entanglement convertibility based on intensive works in recent years. The condition for the LOCC entanglement convertibility of two pure entangled states is given by the Nielsen's theorem [**?**]: The necessary and sufficient condition to convert $|\Psi\rangle$ to $|\Phi\rangle$ by LOCC is that the Schmidt coefficients of $|\Psi\rangle$ are majorized by those of $|\Phi\rangle$. On the other hand, for infinite dimensional systems (or continuous variable systems), there are still many open questions on general LOCC entanglement convertibility, although there are important works [2], which have investigated a limited class of local operations (Gaussian operations), from a practical point of view.

As the first step to understanding the full potential of infinite dimensional systems, we study how the results obtained for LOCC entanglement convertibility in finite dimensional systems can be generalized for LOCC settings in infinite dimensional systems. In this paper, we deal with a general LOCC setting, which includes only countably infinite measurement results for simplicity. We give a complete proof of Nielsen's theorem in infinite dimensional systems and then show a new classification of entangled states, which have infinite Schmidt rank, based on the generalized Nielsen's theorem.

To prove the sufficient part of the theorem, we first define ϵ-convertibility of LOCC such that $|\Psi\rangle$ is ϵ-convertible to $|\Phi\rangle$ iff for any $\epsilon > 0$, there exist a LOCC Λ such that $\|\Lambda(|\Psi\rangle\langle\Psi|) - |\Phi\rangle\langle\Phi|\| < \epsilon$. Then, we prove that for any $\epsilon > 0$, the sufficient condition for ϵ-convertibility is that the Schmidt coefficients of $|\Psi\rangle$ are majorized by those of $|\Phi\rangle$. For the necessity part, we show a new simple derivation of the Lo-Popescu's theorem in infinite dimensional systems. This completes the proof.

In finite systems, the Schmidt rank (the rank of reduced density matrix) gives the necessary condition of LOCC convertibility. However in infinite dimensional systems classification by Schmidt rank is not useful since almost all states have infinite Schmidt ranks. From the infinite dimensional version of Nielsen's theorem, we present a new entanglement monotone which can be applied to entangled states

having infinite Schmidt rank. This monotone can be considered an extension of the Schmidt rank.

Since the (usual) Schmidt rank represents how fast the Schmidt coefficients disappear, it is quite natural to consider that this extended Schmidt rank represents how fast the Schmidt coefficients converge to zero. For a bipartite state $|\Psi\rangle$ for which the Schmidt coefficients are given by $\{\lambda_n\}_{n=1}^{\infty}$, there exists a natural number $s(\Psi) > 1$ such that for $r > s$, $\lim_{n\to\infty} n^r \lambda_n = \infty$ for $r \leq s$, $\lim_{n\to\infty} n^r \lambda_n = 0$ and if for all $r \in N$ $\lim_{n\to\infty} n^r \lambda_n = 0$ is valid, define $s = \infty$. Using this definition and the generalized Nielsen's theorem, we can show that the necessary condition to convert $|\Psi\rangle$ to $|\Phi\rangle$ by LOCC is $s(|\Psi\rangle) \leq s(|\Phi\rangle)$. We can also extend $s(\Psi)$ to real numbers by our definition. In this case, the monotonicity in terms of entanglement is also preserved. Further, we investigate SLOCC [3] convertibility of states in infinite dimensional space using $s(\Psi)$. The monotone $s(\Psi)$ plays crucial role in SLOCC convertibility in a similar way to the finite dimensional case.

References

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