# Polarisation entanglement from a symmetric solid state photon source

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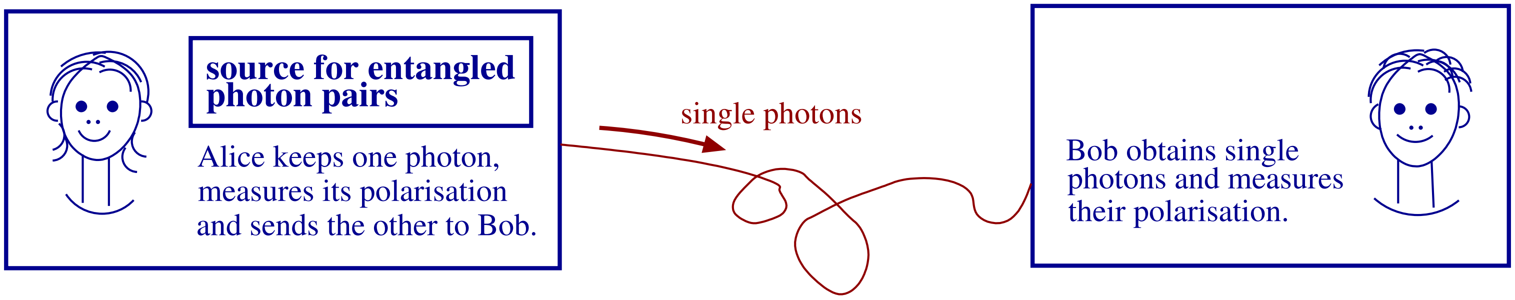
# Polarisation entanglement for quantum cryptography

**Aim:** to enable Alice and Bob to establish a **completely random sequence of bits** that is only known to them — **a secret key**

**Possible realisations:** [1,2]



**source for entangled photon pairs**

Alice keeps one photon, measures its polarisation and sends the other to Bob.

single photons

Bob obtains single photons and measures their polarisation.

If both measure the same basis, the polarisations are **highly correlated**. About every second measurement leads to one new bit of the secret key.

[1] Ekert, PRL **67**, 661 (1991).

[2] See also protocol by Bennett and Brassard (1984).

# Already existing photon sources

**Single photons:**

- on demand: atom-cavity schemes [1]
- quantum dots [2]
- NV color centres in a diamond [3]

**Entangled photon pairs:**

- atomic cascades [4]
- parametric down conversion [5]

[1] Kuhn, Hennrich, Rempe, PRL **89**, 067901 (2002).

[2] Kim, Benson, Kan, Yamamoto, Nature **397**, 500 (1999).

[3] Kurtsiefer, Mayer, Zarda, Weinfurter PRL **85**, 290 (2000);
Beveratos, Brouri, Gacoin, Villing, Poizat, Grangier, PRL **89**, 187901 (2002).

[4] Aspect, Grangier, Roger, PRL **49**, 91 (1982).

[5] Kwiat, Mattle, Weinfurter, Zeilinger, Sergienko, Shih, PRL **75**, 4337 (1995).

# Proposal for a new photon source

**Common believe:**

Photons can only be entangled if they have been created by the **same source**, as in parametric down conversion.

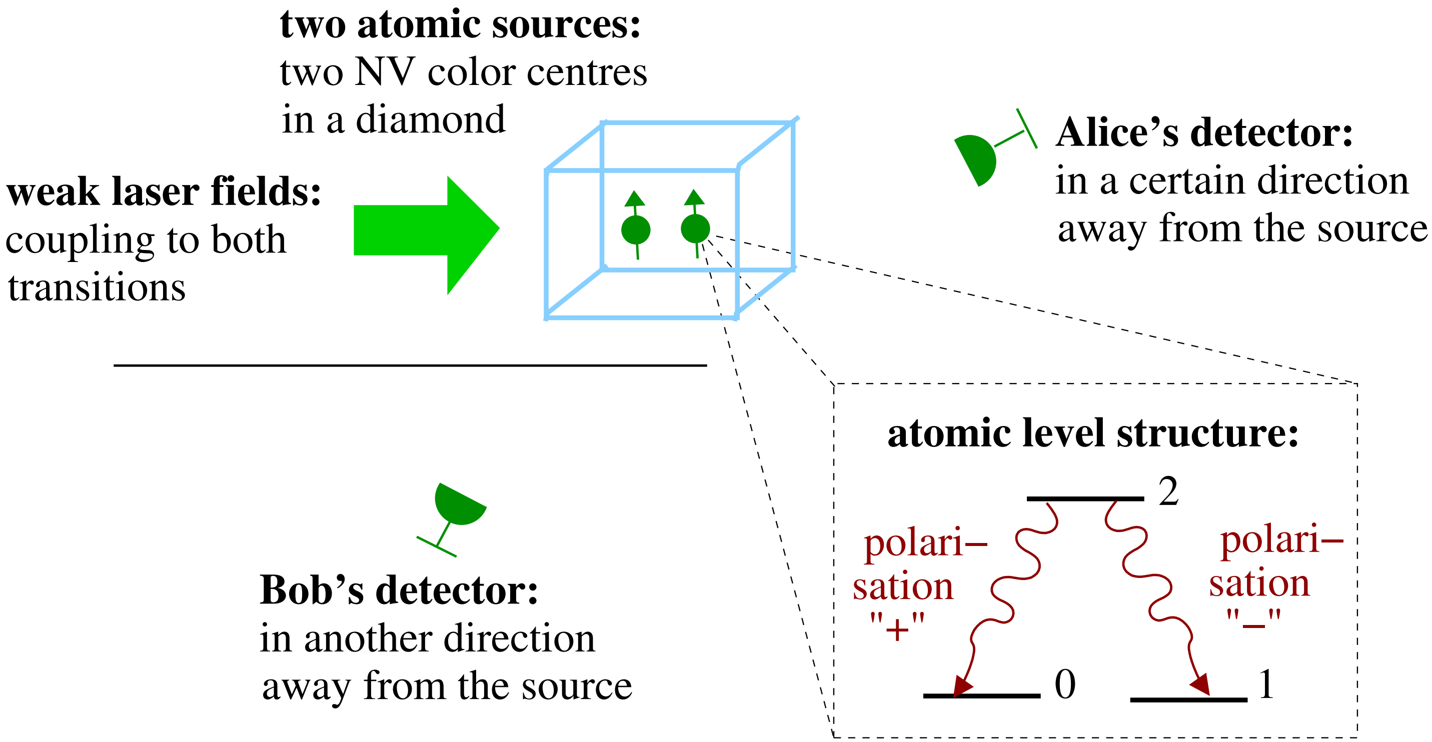
**New observation:**

Entanglement is also obtainable from **different sources** if **several sources contribute** to the detection of **single photons**.

Here: **Interference effects** exclude all paths where the photons have the same polarisation:

**symmetry of the setup** $\Rightarrow$ **maximally entangled photon pairs**

# Proposed experimental setup:



**two atomic sources:**
two NV color centres
in a diamond

**weak laser fields:**
coupling to both
transitions

**Alice's detector:**
in a certain direction
away from the source

**Bob's detector:**
in another direction
away from the source

**atomic level structure:**

If Alice and Bob, both, detect a photon **within a time interval** $\Delta t$, then these photons are **maximally entangled** in polarisation.

**Main disadvantage:**

- **The scheme is probabilistic:**

  Alice and Bob have to compare the arrival times of detected photons via classical communication to identify the maximally entangled pairs.

**Advantages:**

- **Antibunching** between entangled photon pairs is guaranteed.
- **A stable laser source is not required:** Rabi frequency and frequency of the laser can even vary in time.
- **Relatively simple and robust experimental setup** (for example: two NV color centres in a diamond or trapped atoms)

# The creation of entanglement

**Hamiltonian:** $H = \hbar \sum_{i=1,2} \sum_{j=0,1} \sum_{\mathbf{k},\lambda} g_{\mathbf{k},\lambda}^{(j)*} \, \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}_i} \, \mathrm{e}^{-\mathrm{i}(\omega_0-\omega_k)t} \, a_{\mathbf{k},\lambda}^{\dagger} \, |j\rangle_{ii}\langle 2| + \mathrm{h.c.}$

**Reset operator for single photon excitation:** [1,2]

- Excitation of a single photon in the mode $(k_0\hat{\mathbf{k}}, \lambda)$ corresponds to a change of the **state of the source from** $|\psi\rangle$ **to** $R_{\hat{\mathbf{k}}\lambda} |\psi\rangle$ with

$$R_{\hat{\mathbf{k}}\lambda} = \sum_{i=1,2} \sum_{j=0,1} \mathrm{constant} \cdot \left( \hat{\mathbf{D}}_{2j} \cdot \epsilon_{\hat{\mathbf{k}}\lambda} \right) \mathrm{e}^{-\mathrm{i}k_0\hat{\mathbf{k}}\cdot\mathbf{r}_i} \, |j\rangle_{ii}\langle 2|$$

- happens with the probability density $I_{\hat{\mathbf{k}}\lambda}(\psi) = \| R_{\hat{\mathbf{k}}\lambda} |\psi\rangle \|^2$

[1] Schön, Beige, PRA **64**, 023806 (2001); Beige, Schön, Pachos, Fort. Phys. **50**, 594 (2002).

[2] Beige, Pachos, Walther, PRA **66**, 063801 (2002).

**Basic idea:**

We choose the **position of Alice and Bob** such that

$$R_{\hat{\mathbf{k}}\lambda}^{\text{Alice}} \equiv \sum_{j=0,1} \text{constant} \cdot \left(\mathbf{D}_{2j} \cdot \epsilon_{\hat{\mathbf{k}}\lambda}\right) \left( |j\rangle_{11}\langle 2| + |j\rangle_{22}\langle 2| \right)$$

$$R_{\hat{\mathbf{k}}\lambda}^{\text{Bob}} \equiv \sum_{j=0,1} \text{constant} \cdot \left(\mathbf{D}_{2j} \cdot \epsilon_{\hat{\mathbf{k}}\lambda}\right) \left( |j\rangle_{11}\langle 2| - |j\rangle_{22}\langle 2| \right)$$

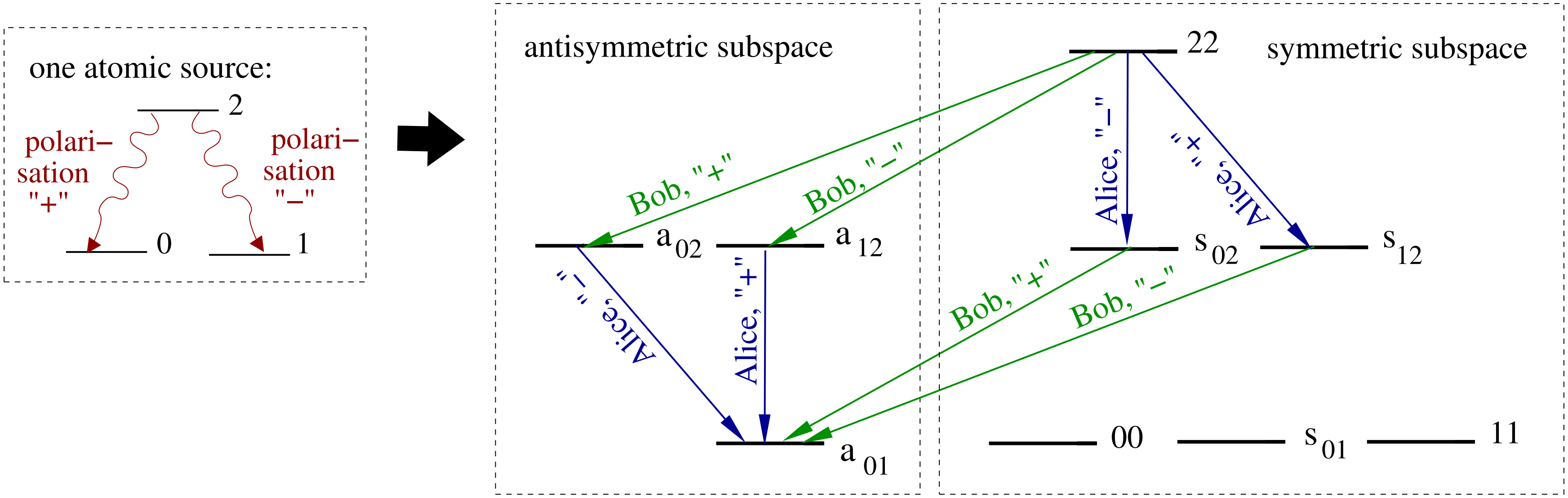
- A photon detection by Bob **changes the symmetry** of the source.
- If Alice receives a photon, the **symmetry of the source does not change**.

**Dicke states:**

$$|s_{jk}\rangle \equiv \left( |jk\rangle + |kj\rangle \right) / \sqrt{2}$$

$$|a_{jk}\rangle \equiv \left( |jk\rangle - |kj\rangle \right) / \sqrt{2}$$

# Combined level scheme of the atomic source:



# Creation of a maximally entangled photon state if:

- Both, Alice and Bob, detect a photon within a time interval $\Delta t$.
- Laser excitation can be neglected within $\Delta t$ ($\sim$ antibunching).
- The positions of the detectors are chosen carefully.

## Example:

- **intital state:** $|\psi\rangle|0_{\rm ph}\rangle$
- **Bob** receives the **first photon:** $\longrightarrow \quad \sum_\lambda R^{\rm Bob}_{\hat{\mathbf{k}}\lambda}|\psi\rangle\,|1_{{\rm Bob},\lambda}\rangle/\|\cdot\|$
- **Alice** receives a **second photon:**

$$\longrightarrow \quad \sum_{\lambda'}\sum_{\lambda} R^{\rm Alice}_{\hat{\mathbf{k}}\lambda'}\, R^{\rm Bob}_{\hat{\mathbf{k}}\lambda}|\psi\rangle\,|1_{{\rm Alice},\lambda'}\rangle\,|1_{{\rm Bob},\lambda}\rangle/\|\cdot\|$$

$$= \Big[\left(\hat{\mathbf{D}}_{20}\cdot\epsilon^{\rm Alice}_{\hat{\mathbf{k}}+}\right)\left(\hat{\mathbf{D}}_{21}\cdot\epsilon^{\rm Bob}_{\hat{\mathbf{k}}-}\right)|1_{{\rm Alice},+}\rangle\,|1_{{\rm Bob},-}\rangle$$

$$-\left(\hat{\mathbf{D}}_{21}\cdot\epsilon^{\rm Alice}_{\hat{\mathbf{k}}-}\right)\left(\hat{\mathbf{D}}_{20}\cdot\epsilon^{\rm Bob}_{\hat{\mathbf{k}}}\right)|1_{{\rm Alice},-}\rangle\,|1_{{\rm Bob},+}\rangle\Big]|a_{01}\rangle/\|\cdot\|$$

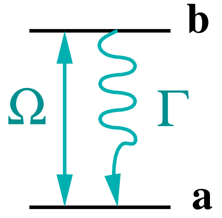
The detection of a "+" photon **should be as likely as** the detection of polarisation "-".

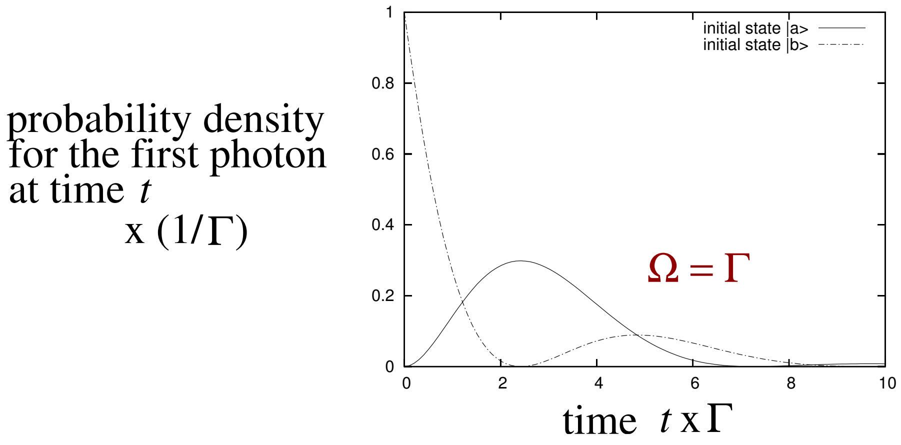
# Main sources for errors

## Emission from unwanted states within $\Delta t$ due to laser excitation:

The laser fields might reexcite states within $\Delta t$. This might result in the detection of **non-entangled photons**.

laser driven
two–level system:

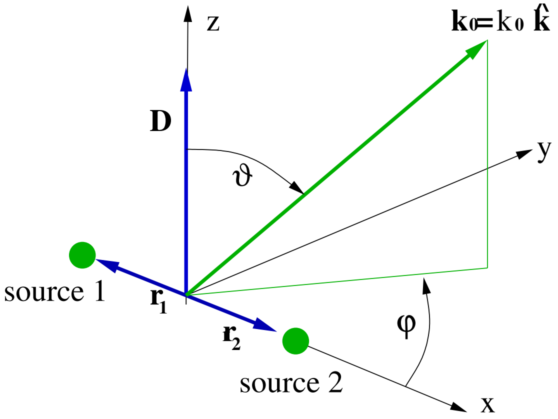




This error is small if the **laser excitation** is chosen **relatively weak**.

## Emission from unwanted states within $\Delta t$ due to finite detector size:

Alice and Bob are placed in the minima and maxima of a quantum mechanical double slit experiment with two two-level atoms: [1]



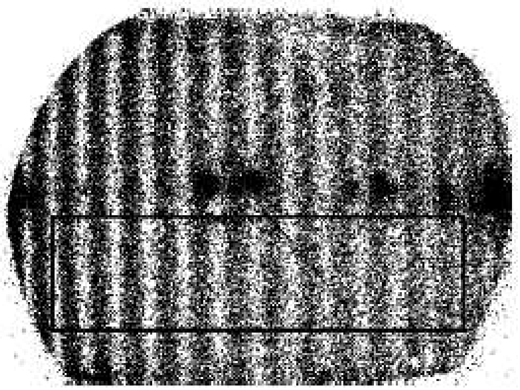


FIG. 5. Experimental fringe data

This error is in general small if the **distance between the atomic sources** is **relatively small** (of the order of a several wavelengths $\lambda_0$).

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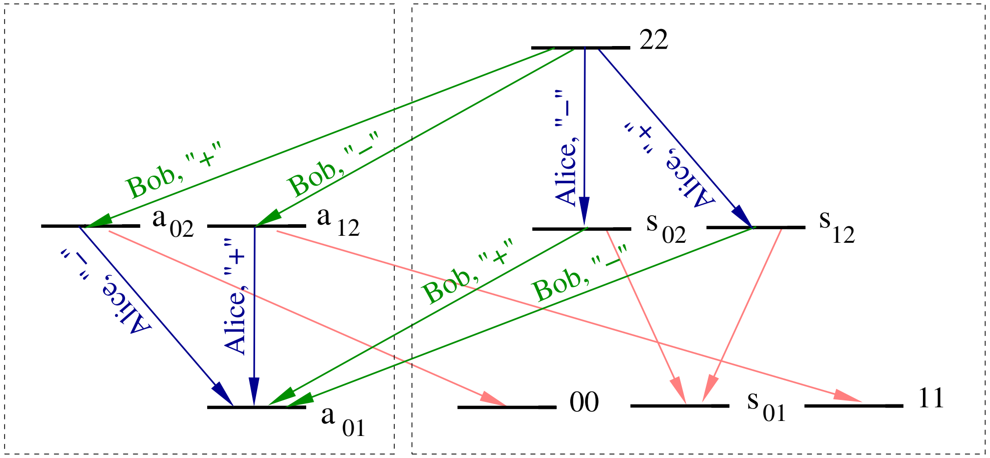
[1] Eichmann, Berquist, Bollinger, Gilligan, Itano, Wineland, PRL **70**, 2359 (1993).

# Efficiency of the source

**The relative number of useful photons and their rate depends on:**

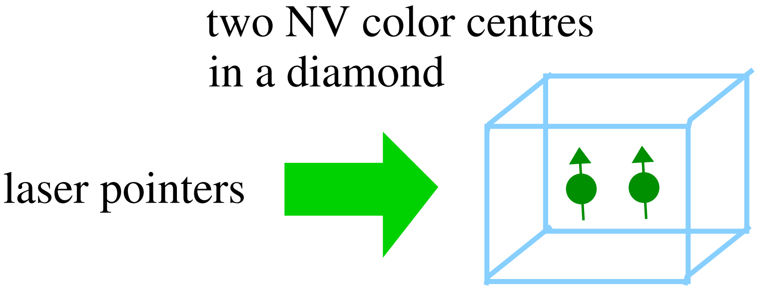
- the **steady state population** in $|22\rangle$:



- the **minimum rate of non entangled pairs** that Alice and Bob accept (higher rates allow to increase detector sizes and laser strength)
- the **distance of the sources** and the **size of the detectors**

# Final remarks

We proposed a new **solid state** photon source for **quantum cryptography**:



- **Polarisation entanglement** is obtained via **interference effects**.

- The setup is **robust against parameter fluctuations**...
  ... and hopefully **relatively easy to realise**.

- Quantum cryptography with this source requires a lot of classical communication but **no storage of photons**.

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