Violation of Bell inequalities
and quantum tomography
with pure-states,
Werner-states & MEMS
created by
a universal quantum entangler

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Polarization entangled photon states

- Non-collinear type II phase matching

(Kwiat et al. PRL '95)

\[ |\Psi| = 1/\sqrt{2} \left( |H_A \rangle |V_B \rangle + e^{i\phi} |V_A \rangle |H_B \rangle \right) \]

\( \phi = 0: \) triplet state; \( \phi = \pi: \) singlet state

- Two type I crystal geometry

(Kwiat et al. PRA '99)

\[ |\Phi| = 1/\sqrt{2} |H_A \rangle |H_B \rangle + e^{i\phi} |V_A \rangle |V_B \rangle \]
Our high brilliance source of polarization entanglement

1) All couples of degenerate entangled photons accessible to the detection apparatus
2) Test of Bell’s inequalities performed over the whole emission cone

|Φ⟩ = 1/√2 |H⟩ |H⟩ + e^{iφ} |V⟩ |V⟩

(Giorgi et al. Las. Phys. ’03)
Experimental setup

Single arm interferometer

Entanglement ring

Phase controlled by micrometric displacement $\Delta d$

$\Delta \phi = \pi \iff \Delta d \sim 60 \, \mu m$
Polarization entanglement (over the whole emission cone)

$|\Phi_\rightarrow = 1/\sqrt{2}(|H,H\rangle - |V,V\rangle)$

Test of Bell-CHSH inequality:

$|S| = 2.5564 \pm .0026 \rightarrow 213-\sigma$ violation
Brightness and robustness of entanglement

- more than $2 \cdot 10^5 \, \text{s}^{-1}$ entangled photon pairs generated over the entire emission cone at $\lambda = 2\lambda_p$ ($P \sim 50 \, \text{mW}$)
Ou-Mandel interferometry test

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} |H\rangle |V\rangle + e^{i\phi} |V\rangle |H\rangle \]
Entanglement source: characteristics

- single arm interferometer $\rightarrow$ ultrastable
- high brightness
- generation of pure entangled states
- spatial characteristics $\rightarrow$ make possible the direct generation of any kind of bi-partite quantum state

Applications to quantum communications: teleportation, quantum cryptography.....

Possible implementation of several schemes of quantum information and communication
Mixed States

Interaction with the enviroment → Noise → Decoherence

\[ \rho_{in} = |\Psi_+\rangle\langle\Psi_+| \rightarrow \rho_{out} = p |\Psi_+\rangle\langle\Psi_+| + \{(1-p)/4\}.I \]

Werner state
- Presence of noise responsible of the depletion of correlation within photon pairs.

- Noise level may be so high that, in spite of the fact that some degree of entanglement survives, the state is no longer non local.

**Werner state:**

\[
p > \frac{1}{3} \quad \text{entangled}
\]
\[
p \leq \frac{1}{3} \quad \text{separable}
\]

\[
p > \frac{1}{\sqrt{2}} \quad \text{non locality: Bell-CHSH violation}
\]
\[
p \leq \frac{1}{\sqrt{2}} \quad \text{locality: no Bell-CHSH violation}
\]

\[
p \in \left(\frac{1}{3}, \frac{1}{\sqrt{2}}\right) \quad \text{entangled, no Bell-CHSH violation}
\]

Werner, PRA '89
Generation of mixed states

(Barbieri et al. quant/ph-0303018)

Patchwork for Werner states:

1. \( \frac{1}{\sqrt{2}}(|H,H> - |V,V>) \rightarrow \frac{1}{\sqrt{2}}(|H,V> - |V,H>) \)

2. Thickness of glass \( G > \) photon wavepacket to induce decoherence

3. \( \frac{1}{\sqrt{2}}(|H,V> - |V,H>) \rightarrow \frac{1}{\sqrt{2}}(|H,H> - |V,V>) \)
E-ring partition for Werner states

\[ A \iff |\Psi_+><\Psi_-| \to p \]

\[ B \iff |H,V> - |V,H> \]

\[ C \iff |H,H> - |V,V> \]

\[ B = C \]

\[ B + C \iff \frac{1}{4}(|HV><HV| + |VH><VH| + |HH><HH| + |VV><VV|) = \frac{1}{4} I \]

\[ \to 1-p \]
\[ \rho = |\Psi_-><\Psi_-| \]

\[ \rho = \frac{1}{2} \left[ |\Psi_-><\Psi_-| + |\Phi_-><\Phi_-| \right] \]

\[ \rho = \frac{1}{4} \left[ |HV><HV| + |VH><VH| + |HH><HH| + |VV><VV| \right] \]

\[ p = 0 \]
\[ \rho = p|\Psi_+\rangle\langle\Psi_-| + \frac{(1-p)}{4}I \]

\[ P=0.82 \]

\[ P=0.59 \]

\[ P=0.47 \]

\[ P=0.27 \]
Bell inequalities for Werner states

Non locality

Local realism

$1/\sqrt{2}$
\[ T(\rho) = (\max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\})^2 \]
\[ \lambda_i : \text{square roots of the eigenvalues} \]
\[ S_L = 4/3[1 - \text{Tr}(\rho^2)] = 1 - \rho^2 \]
Maximally Entangled Mixed States (MEMS)

\[
\rho_{\text{MEMS}} = \begin{pmatrix}
1-2g(p) & 0 & 0 & 0 \\
0 & g(p) & -p/2 & 0 \\
0 & -p/2 & g(p) & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\{ g(p) = p/2 \quad p \geq 2/3 \\
g(p) = 1/3 \quad p > 2/3
\}

\[
p = 0: \begin{pmatrix}
1/3 & 0 & 0 & 0 \\
0 & 1/3 & 0 & 0 \\
0 & 0 & 1/3 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
p = 1: \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1/2 & -1/2 & 0 \\
0 & -1/2 & 1/2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

W. J. Munro et al. PRA '01
Patchwork for MEMS

\[ p \geq \frac{2}{3} \]

3. \( |\Phi_-<\Phi_-| \rightarrow |\Psi_-<\Psi_-| \) in B and C sections

4. \( |V,V><V,V| \) deleted in the A section, left cone

\[ p < \frac{2}{3} \]

3. \( |\Phi_-<\Phi_-| \rightarrow |\Psi_-<\Psi_-| \) in B and C sections

4. \( |V,V><V,V| \) deleted in the A section, left cone

2. Insertion of glass introduces decoherence on section D, left cone
MEMS

\begin{align*}
p &= 0.68 \\
p &= 0.31 \\
p &= 0
\end{align*}
Experimental realization of Entanglement Witness

\[ W = \frac{1}{2} [ |H,H><H,H| + |V,V><V,V| + |D,D><D,D| + |D^\perp,D^\perp><D^\perp,D^\perp| - |L,R><L,R| - |R,L><R,L| ] \]  

(for Werner states)

\[ |D> \equiv (|H> + |V>)/\sqrt{2} \]

\[ |D^\perp> \equiv (|H>-|V>)/\sqrt{2} \]

\[ |L> \equiv (|H>+i|V>)/\sqrt{2} \]

\[ |R> \equiv (|H>-i|V>)/\sqrt{2} \]

\[ \text{Tr}[W_\rho] \geq 0: \text{ separable} \]

\[ \text{Tr}[W_\rho] < 0: \text{ entangled} \]  

(Guhne et al. PRA '02)
Performed with tunable Werner states, $p \in [0, 1]$

\[
\text{Tr}[W_{\rho_W}] = (1-3p)/4
\]

(Barbieri et al. quant/ph-0307003)
Applications & Perspectives

- Non maximally entangled states, Hardy's ladder proof
- Generalized mixed states with femtosecond pulses
- Amplification of entanglement by synchronous femtosecond pumping: generation of entangled multiphoton states