Implementing Quantum Noiseless Coding

or How to QuWinZip

Masahide Sasaki
Yasuyoshi Mitsumori
Atsushi Hasegawa
Masahiro Takeoka
— Communications Research Lab, Tokyo

Stephen Barnett
Erika Andersson
— U. Strathclyde, Glasgow

John Vaccaro
— UH

Contents

- Review classical noiseless coding
  - – Shannon entropy
- Quantum noiseless coding
  - - von Neumann entropy
- Linear optical schemes
  - - single photon – 3 qubit circuit
- Experimental results
  - - 3 qubits → 2 qubits → 3 qubits

UH

Classical noiseless coding

Shannon – 1948

IDEA: remove redundancy – make coded message as short as possible

Message of letters: **the quick brown…**

Source probabilities: $P(\mathbf{a}), P(\mathbf{b}), P(\mathbf{c}), \ldots, P(\mathbf{z})$

Shannon Entropy:

$$H = -\sum_{n=a,b,c\ldots} P(n)\log_2 P(n)$$

→ Av. information (bits) carried by each letter

Shannon's noiseless coding th^m^:

Encode message of K letters with

$K \times H$ bits as $K \rightarrow 8$

Code common letters → shortest code symbols

Equally likely letters: **still his heart dared...**

$P(\mathbf{a}) = P(\mathbf{e}) = P(\mathbf{i}) = \ldots = P(\mathbf{t})$

Here

$$H = \log_2 N$$

where N is the size of the alphabet

e.g. if $N = 256$, $H = 8$ bits (no better than ASCII)

→ **No compression possible**

Quantum noiseless coding

Schumacher - 1995

Phys. Rev. A **51**, 2738 (1995)

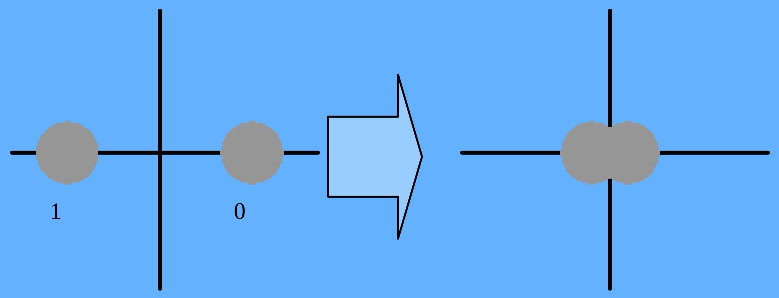
Message of quantum states, e.g.

$$\left|L^{(1)}\right\rangle \otimes \left|L^{(2)}\right\rangle \otimes \left|L^{(3)}\right\rangle \otimes \ldots \left|L^{(K)}\right\rangle$$

If *orthogonal* letters – analysis is *classical*

Long haul communication – weak coherent states – *no longer orthogonal*

e.g.



Probability of each letter: $P_{\mathrm{a}}, P_{\mathrm{b}}, P_{\mathrm{c}}, \ldots$

Average state of a letter:

$$\hat{r} = P_a |L_a\rangle\langle L_a| + P_b |L_b\rangle\langle L_b| + P_c |L_c\rangle\langle L_c| + \ldots$$

Von Neumann entropy

$$S(\hat{r}) = \mathrm{tr}(\hat{r} \log_2 \hat{r})$$

Schumacher's quantum noiseless coding thm

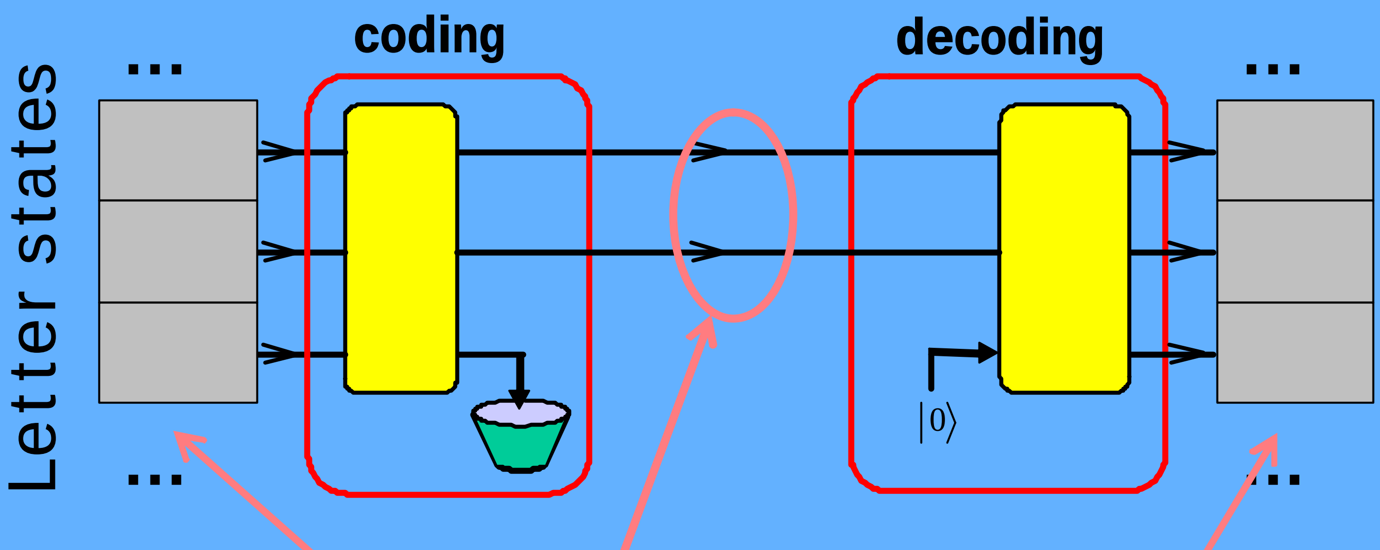
Code blocks of K letters:

$KS(\hat{r})$ qubits are sufficient to encode each block as $K \to \infty$

UH

Method: (1) project onto *most likely* subspace

(2) code message with reduced qubits



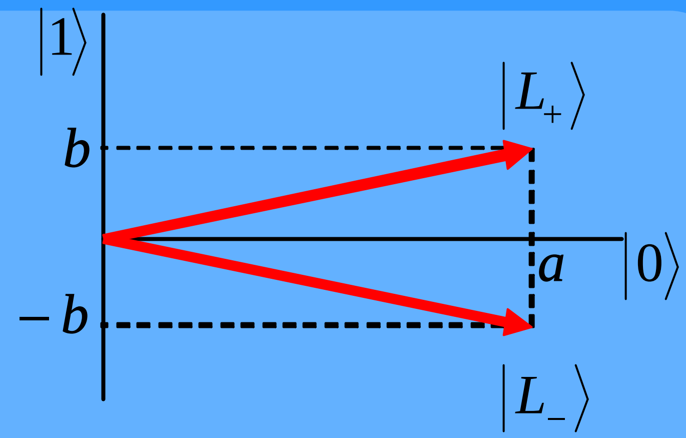
most likely subspace
& reduced qubits

Fidelity: $\text{av. of } \left| \langle \text{original block} | \text{decoded block} \rangle \right|^2$

UH

Jozsa-Schumacher - 1994

J. Mod. Opt. **41**, 2343 (1994)



2 state alphabet:

$$|L_\pm\rangle = a|0\rangle \pm b|1\rangle$$

Special case: equal-likely letter states $P_+ = P_-$

– *not* compressible classically

$$\hat{r} = |a|^2 |0\rangle\langle 0| + |b|^2 |1\rangle\langle 1|$$

$$S(\hat{r}) = -\left(|a|^2 \log_2 |a|^2 + |b|^2 \log_2 |b|^2\right)$$

< 1 for |α|?|β|

Less than 1 qubit to code each letter!

$S(\hat{r}) = 0.469$ for $|a|^2 = 0.9$ **1/2 qubit !**

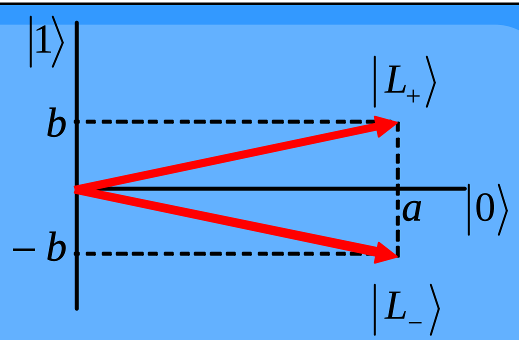
UH

3 qubit example

Jozsa-Schumacher - 1994

$b_n = \pm b$

$b \ll a$



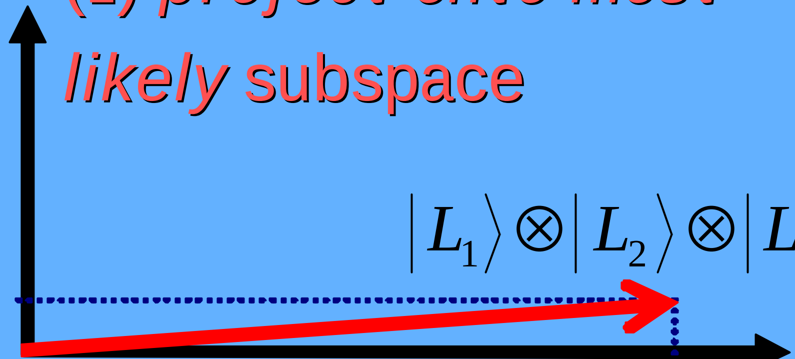
$$|L_1\rangle \otimes |L_2\rangle \otimes |L_3\rangle$$

$$= a^3|000\rangle + a^2 b_3|001\rangle + a^2 b_2|010\rangle + a^2 b_1|100\rangle$$

$$+ \underbrace{a b_2 b_3|011\rangle + a b_1 b_3|101\rangle + a b_1 b_2|110\rangle + b_1 b_2 b_3|111\rangle}_{\text{small weight}}$$

small weight

(1) *project onto most likely* subspace



$$|L_1\rangle \otimes |L_2\rangle \otimes |L_3\rangle$$

nonlinear

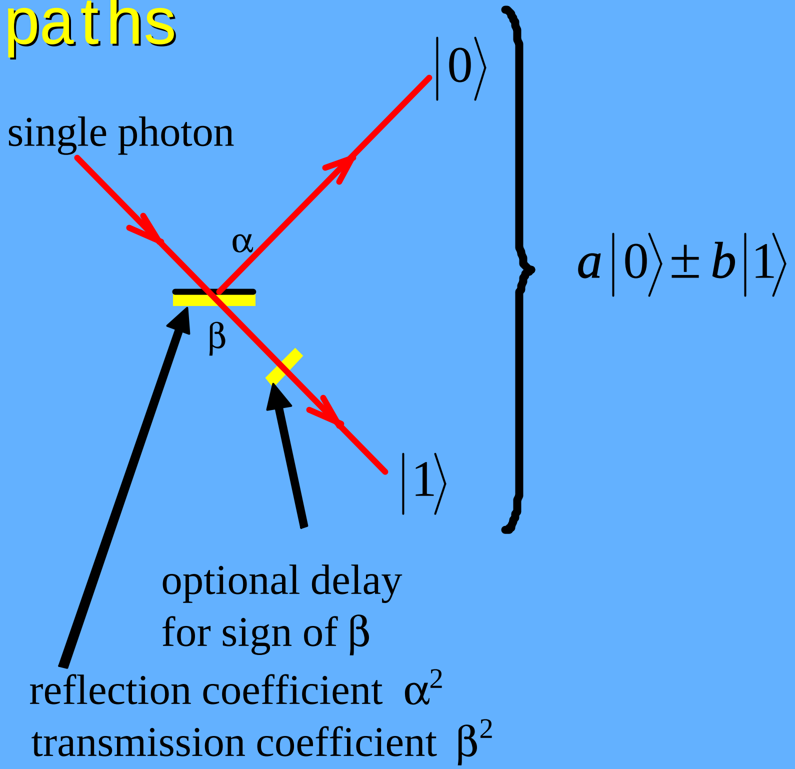
$|000\rangle, |001\rangle, |010\rangle, |100\rangle$

(2) *Encode as* $|00\rangle, |01\rangle, |10\rangle, |11\rangle$

UH

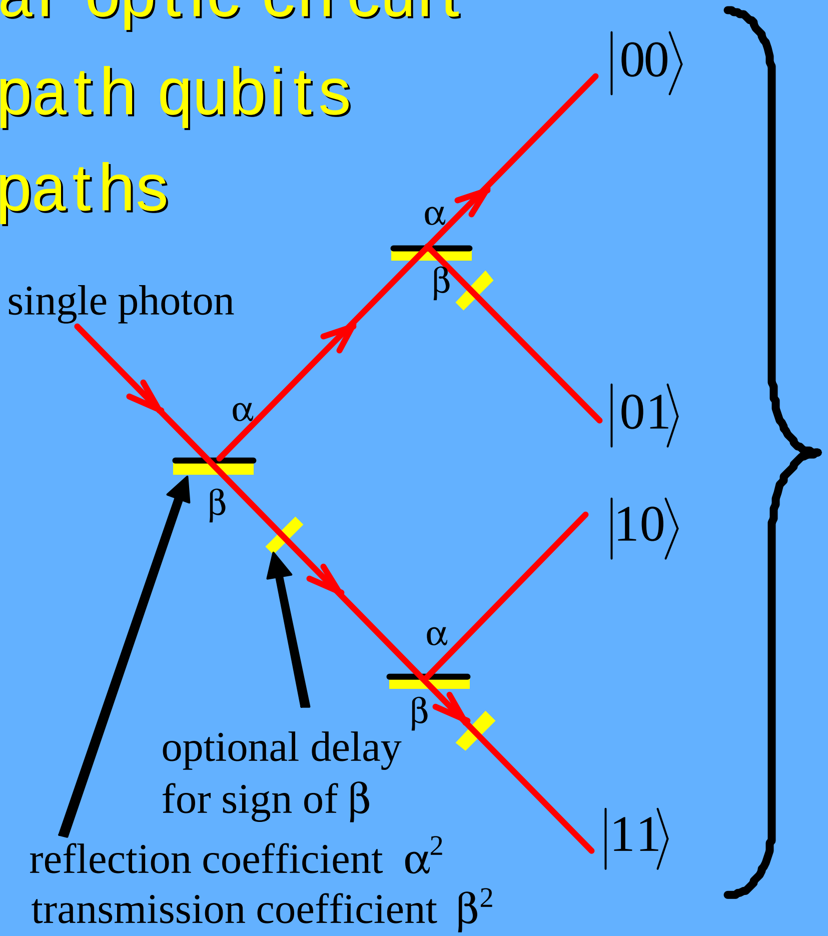
Linear optic circuit

- 3 path qubits
- 8 paths



Linear optic circuit

- 3 path qubits
- 8 paths

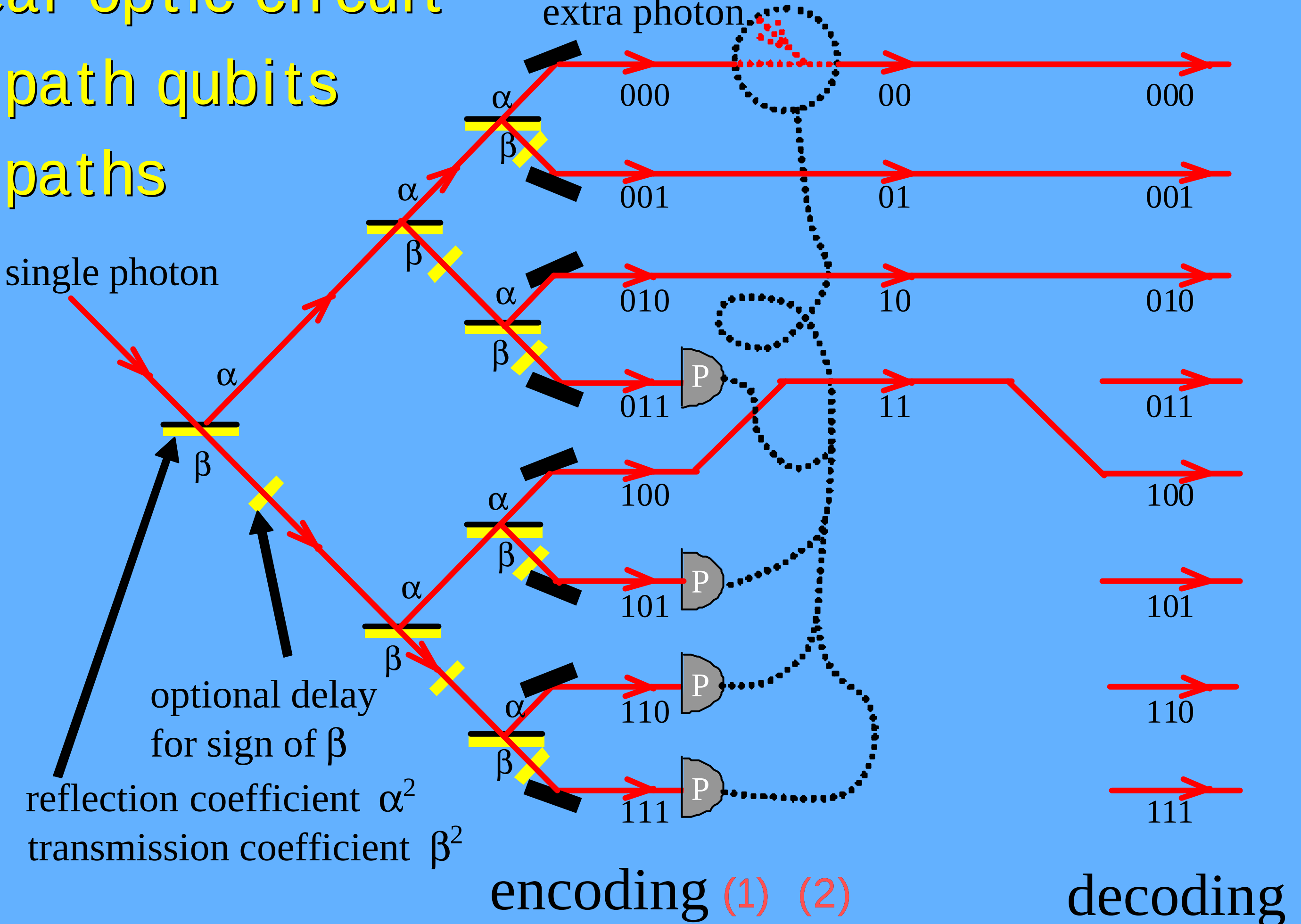


$$\left(a|0\rangle \pm b|1\rangle\right)\otimes\left(a|0\rangle \pm b|1\rangle\right)$$
$$= a^2|00\rangle \pm ab|01\rangle \pm ab|10\rangle + b^2|11\rangle$$

UH

Linear optic circuit

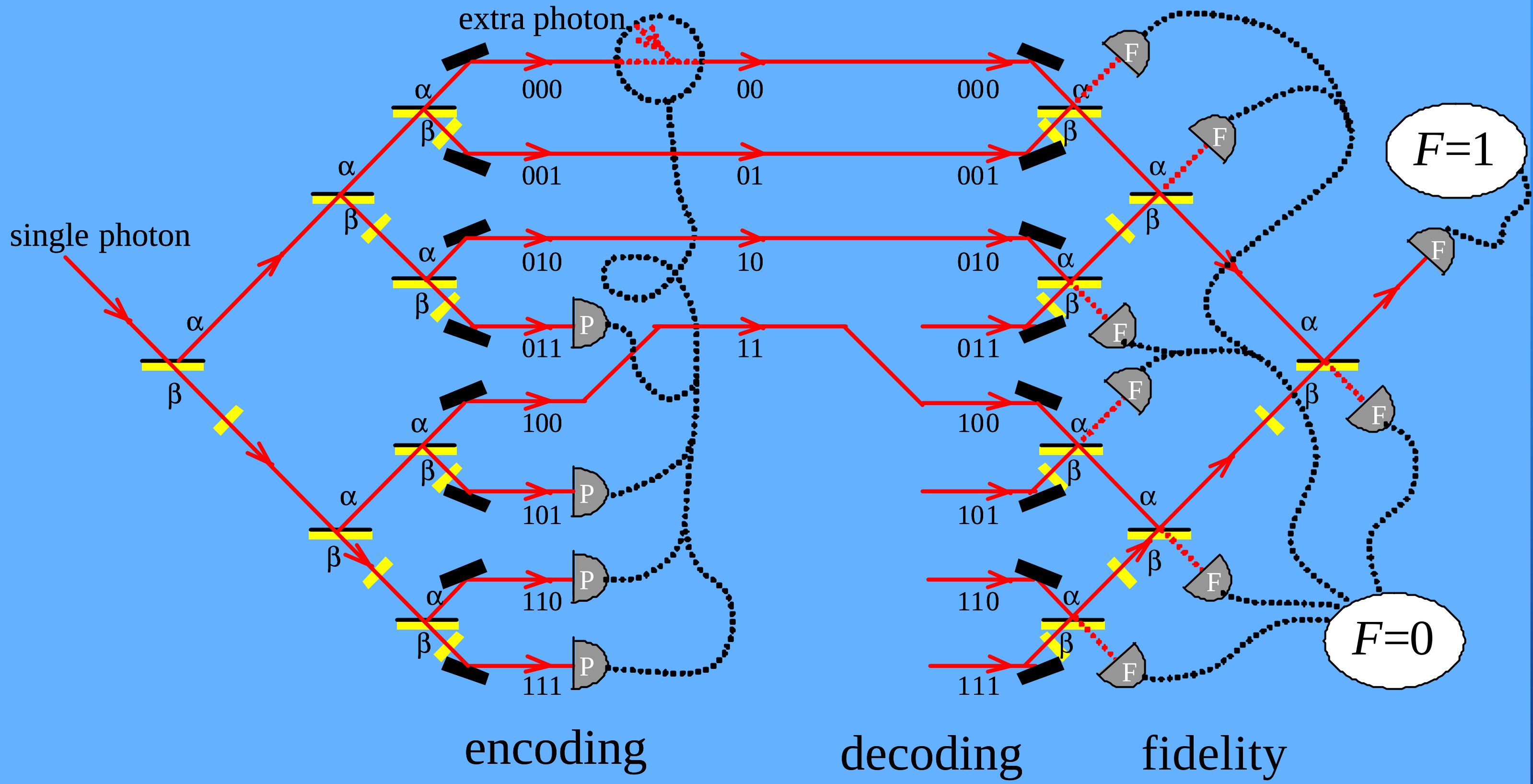
- 3 path qubits
- 8 paths



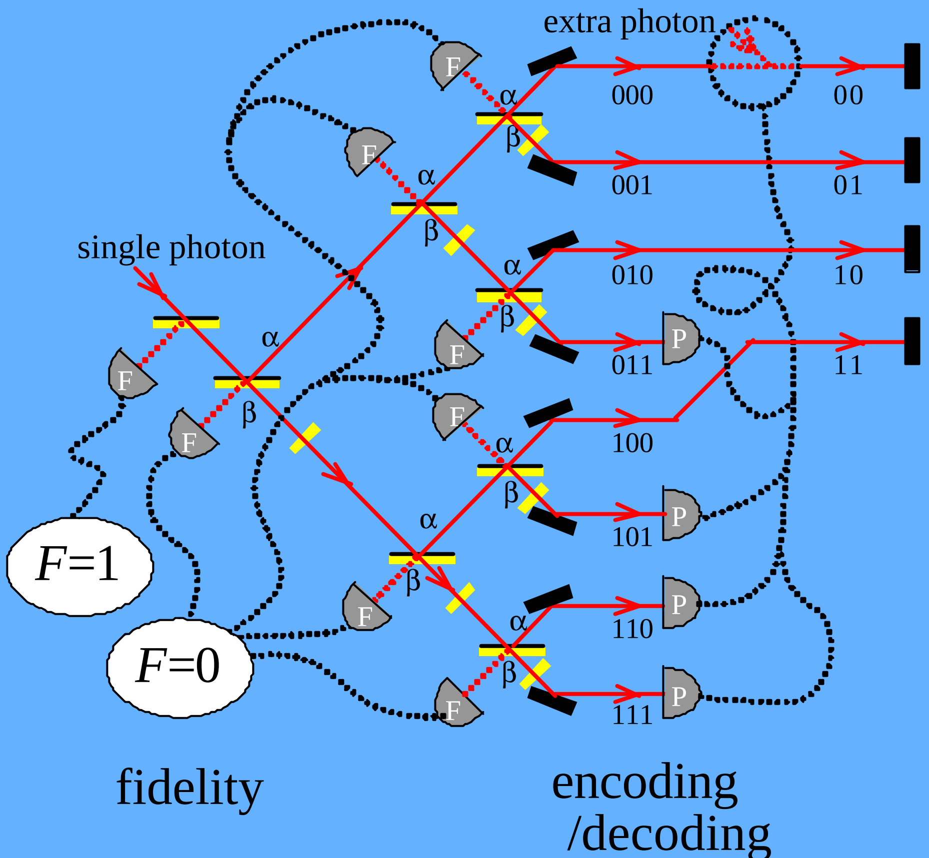
3 qubits 2 qubits 3 qubits

Fidelity = 0.97 0.67 qubits/letter

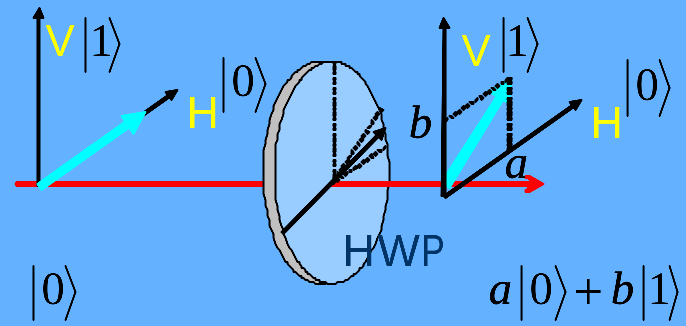
Fidelity measurement



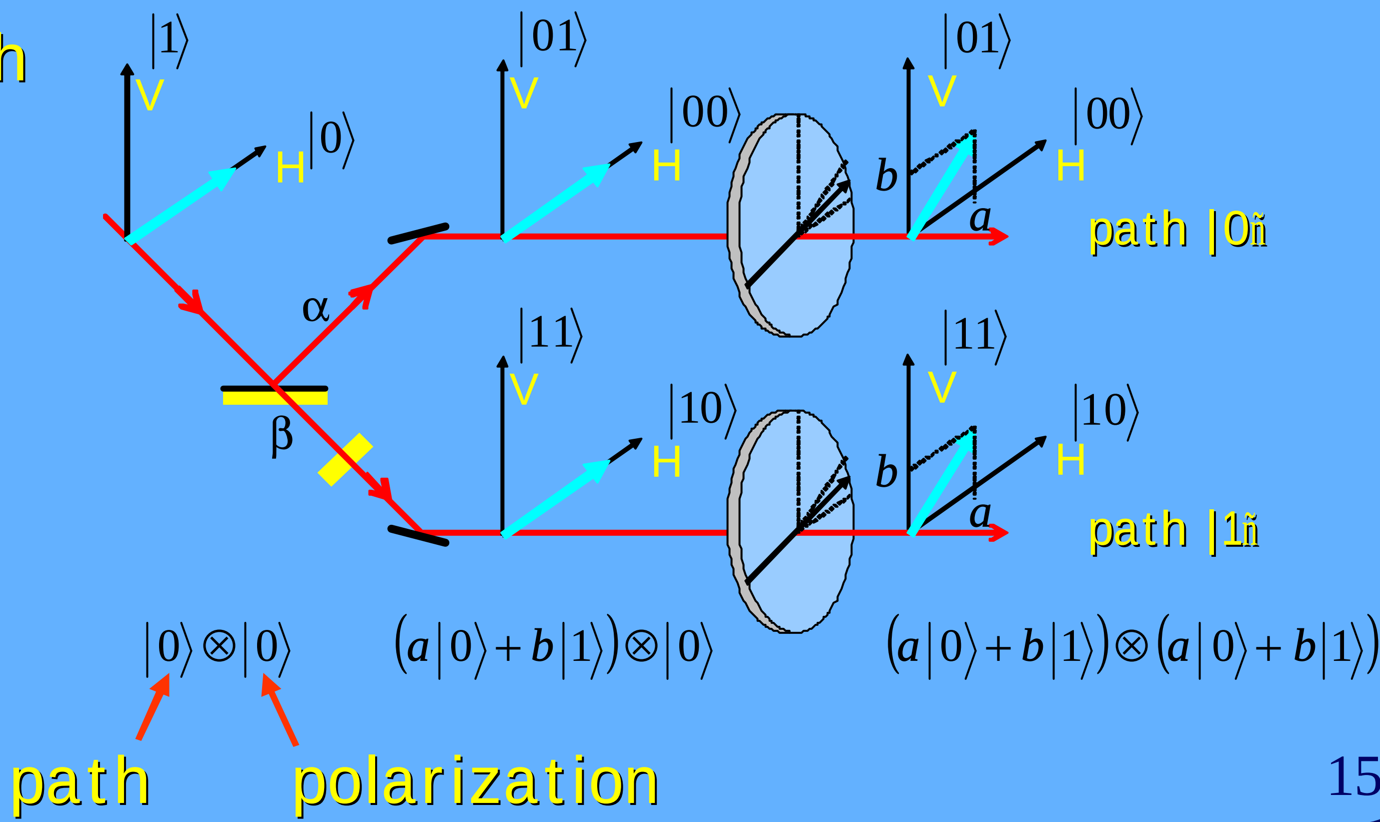
Fidelity measurement



Polarization qubit

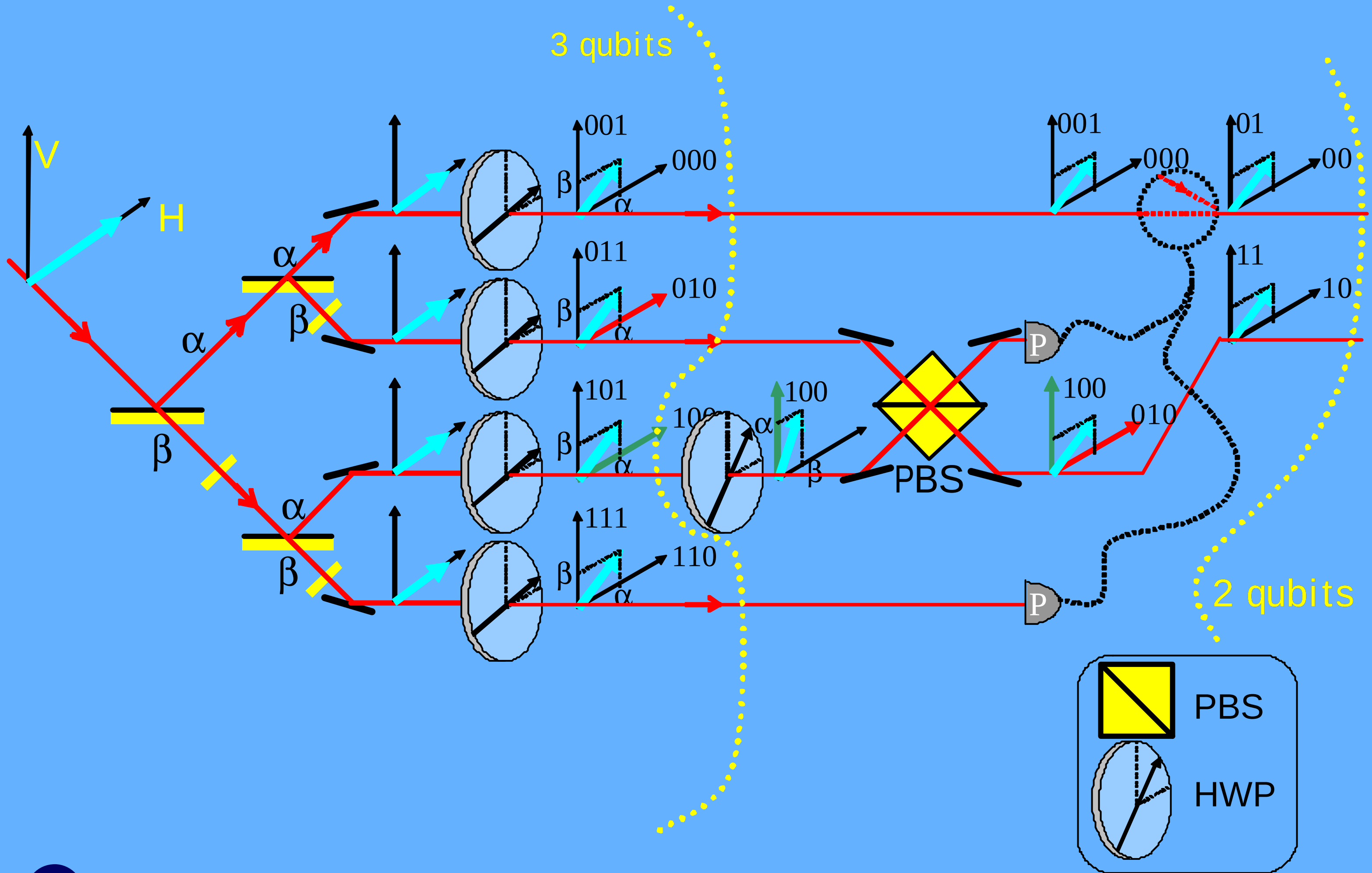


Polarization and path qubits



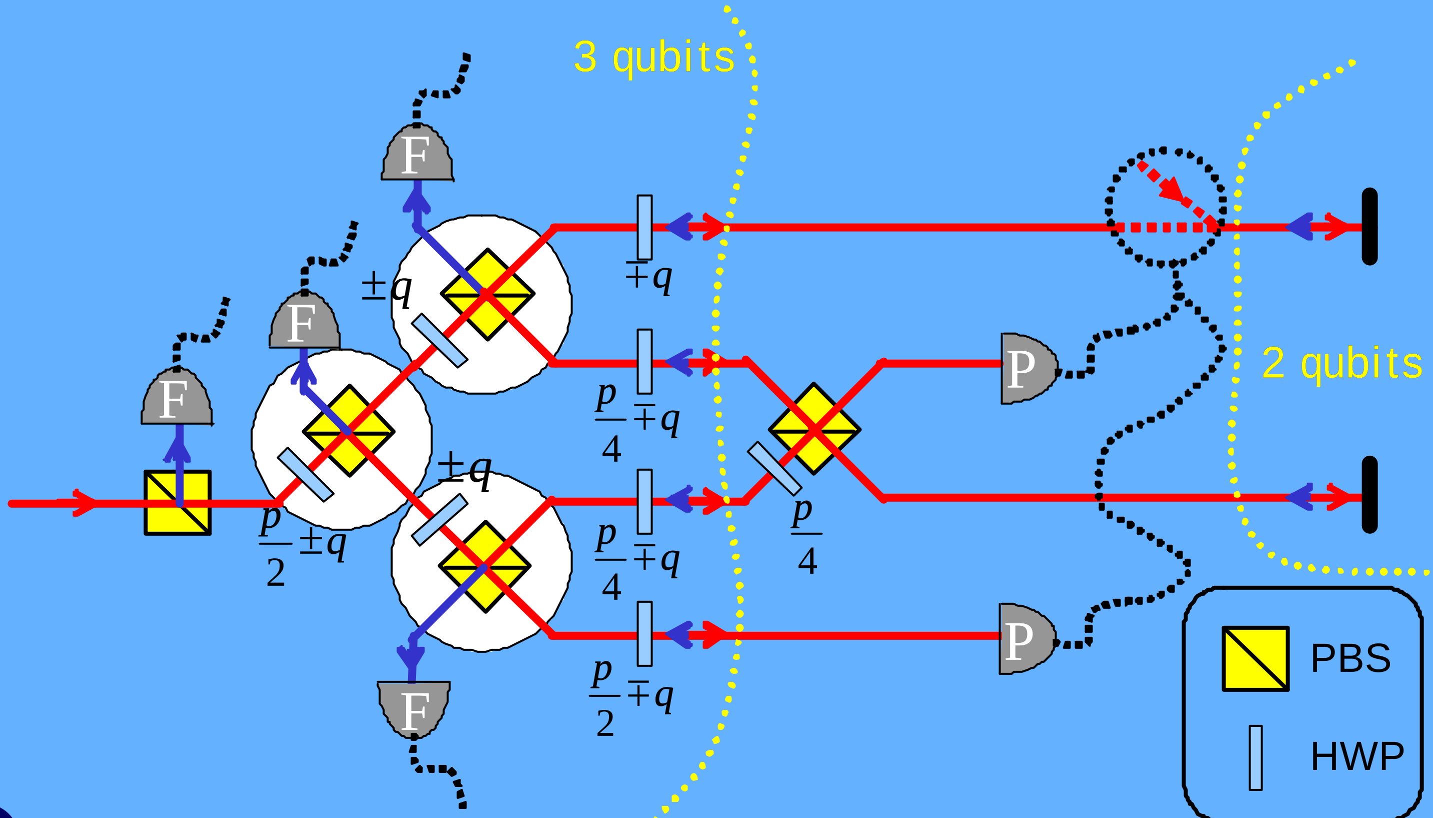
Linear optic circuit

- 2 path qubits, 1 polarization qubit



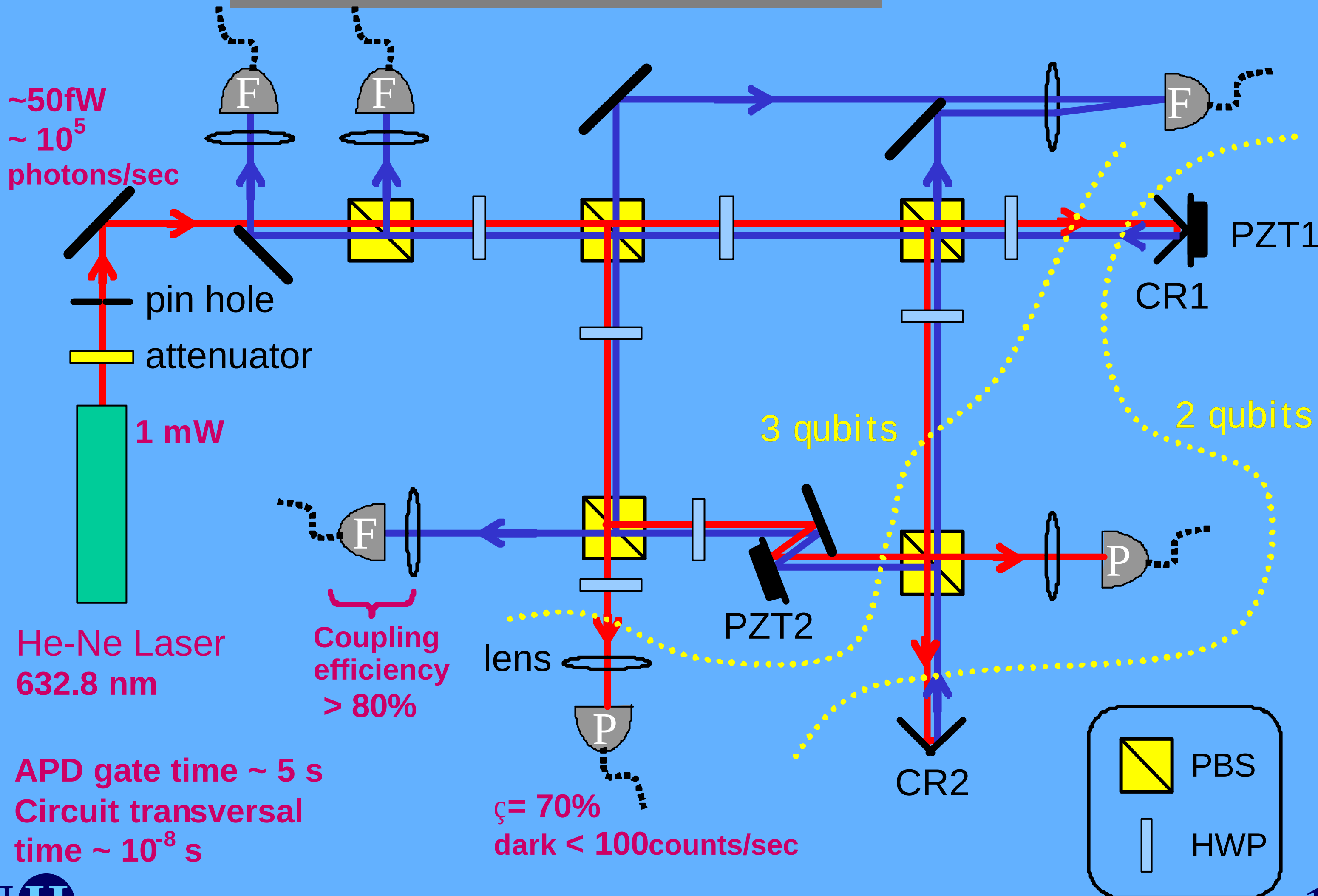
Experimental Implementation

Using PBS to construct the path qubits

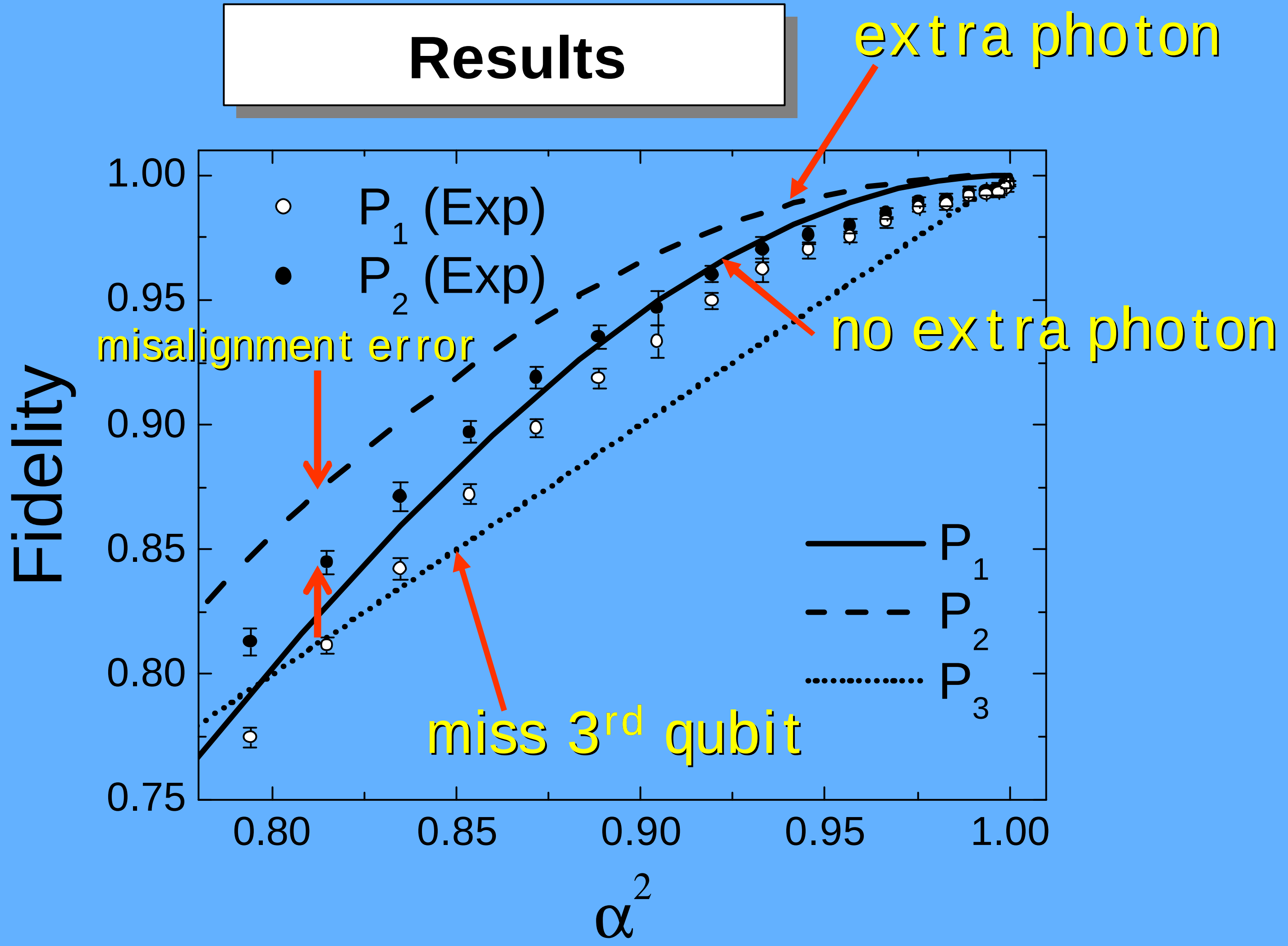


Actual setup

quant-ph/0304036



Results

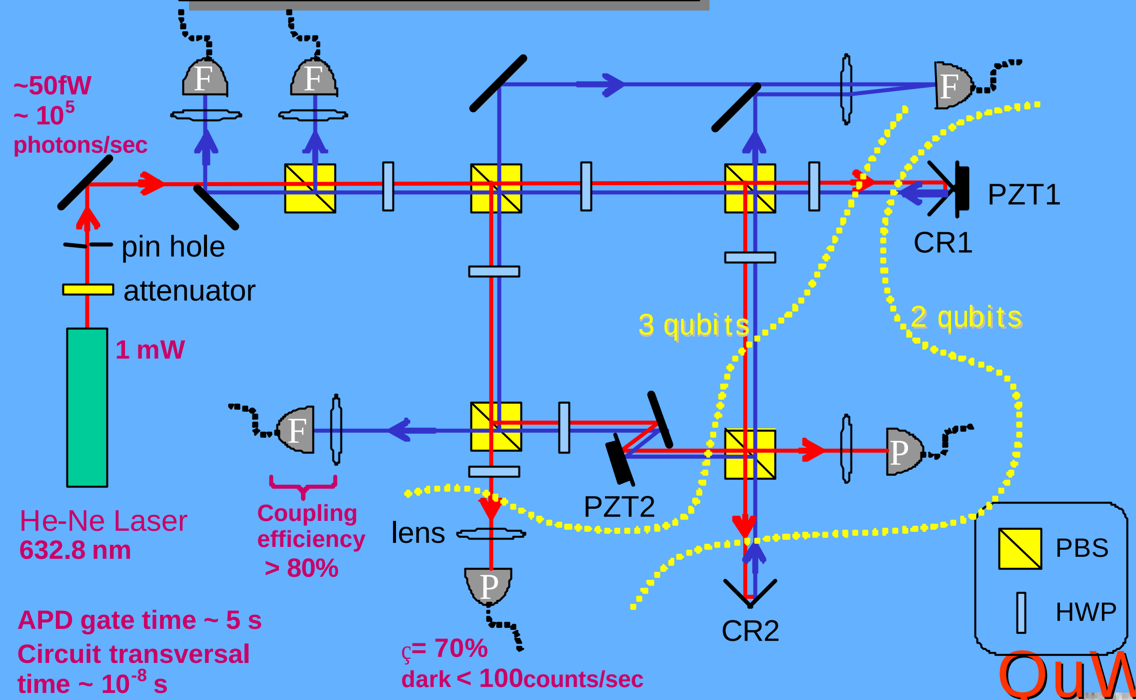


extra photon
no extra photon
misalignment error
miss 3rd qubit
Fidelity
1.00
0.95
0.90
0.85
0.80
0.75
0.80
0.85
0.90
0.95
1.00
α^2
P_1 (Exp)
P_2 (Exp)
P_1
P_2
P_3

Conclusion

Actual setup

quant-ph/0304036



QuWinZip

First *proof-of-principle* demonstration:

3 qubits → 2 qubits → 3 qubits