Spectral decomposition based separability criteria: a numerical survey

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Abstract

A complete characterisation of quantum entanglement [1] has not yet been obtained, the associated separability problem being a difficult one indeed. The goal is to be in a position to assert whether a given state ρ describing a quantum system is useful or not for quantum information processing purposes. For bipartite Hilbert spaces of low dimensionality (d = *N*x*M* = 2x2 and d = 2x3) the Positive Partial Transpose (PPT) criterion is the strongest one, providing a necessary and sufficient condition for quantum separability [2,3]. It is only a necessary criterion for higher dimensionalities. In the present endeavour we revisit the application of different separability criteria based upon the spectral decomposition by following the work reported in [4], i.e., the PPT, reduction [5], majorization [6] and conditional *q*-entropic criteria [7,8], and quantify the relations that link them by means of a Monte Carlo exploration involving the $[(N\text{x}M)^2\text{-}1]$-dimensional space of pure and mixed states distributed according to the Haar measure [9]. The separability chain of implications of the four criterions is discussed with regard to the issue of distillability. We provide numerical evidence by means of our Monte Carlo exploration of the fact that the PPT implies reduction, while reduction implies majorization, and majorization, in turn, implies the conditional q-entropic criteria. We also quantify, for a bipartite system with an arbitrary dimension, the proportion of states ρ that are distillable according to a definite criterion.

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