Feedback in Quantum Communication

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In the classical theory of information transmission through noisy channels, the existence of a feedback channel has been shown not to increase the capacity of a memoryless channel. A memoryless channel is defined as a noisy channel where the noise acts independently on each symbol sent through the channel. The classical feedback channel is presumed to send an exact copy of the received symbol back to the sender, before the next symbol is sent through the noisy channel. Even with this extra information the sender cannot increase the asymptotic rate at which information may be sent. In the broader context of quantum information theory, the exact nature of a feedback channel is a more difficult concept. This is due to the quantum no-cloning theorem, which does not allow an exact copying process to take place, and therefore any such copying process would detract from the fidelity of the received state.

The process of feedback in the quantum scenario would therefore only be useful if it can be shown how *any* quantum information passed from the receiver, Bob, to the sender, Alice, will affect the channel capacities. *Quantum feedback* is therefore defined as a noiseless quantum channel, of arbitrary capacity, from Bob to Alice. The receiver may process the received quantum states in any way, and send an arbitrary amount of quantum or classical information through the feedback channel.

The various classical and quantum capacities of the channel assisted by classical feedback only, may also be considered. However, for the entanglement-assisted feedback capacities the situation is considerably simplified. This is because a noiseless classical feedback channel automatically becomes a noiseless quantum channel, due to teleportation and the availability of an unlimited supply of maximally entangled states.

It shall be shown that the entanglement assisted capacity of a noisy memoryless quantum channel cannot be increased by feedback. Both the entanglement assisted capacity with classical feedback, and the entanglement assisted capacity with quantum feedback, are equivalent to the entanglement assisted capacity without feedback for any memoryless quantum communication channel. Examples of memoryless channels for which the capacities with quantum or classical feedback exceed the unassisted quantum and classical capacities are also given, thus showing that the analogy with the classical theory only holds in the case of the entanglement assisted capacities.