Optimal State Discrimination Using Particle Statistics

S. Bose[1,2], A. Ekert[3], Y. Omar[4,5], N. Paunković[4] and V. Vedral[6]
[1] *Institute for Quantum Information, California Institute of Technology, CA 91125*
[2] *Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*
[3] *Centre for Quantum Computation, DAMTP, University of Cambridge, United Kingdom*
[4] *Centre for Quantum Computation, Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom*
[5] *Centro de Física de Plasmas, Instituto Superior Técnico, P-1049-001 Lisbon, Portugal*
[6] *Optics Section, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom*
(1 April 2003)

We present an application of particle statistics to the problem of optimal ambiguous discrimination of quantum states. The states to be discriminated are encoded in the internal degrees of freedom of identical particles, and we use the bunching and antibunching of the external degrees of freedom to discriminate between various internal states. We show that we can achieve the optimal single-shot discrimination probability using only the effects of particle statistics. We discuss interesting applications of our method to detecting entanglement and purifying mixed states. Our scheme can easily be implemented with the current technology.

Recently, there has been an emerging interest in the use of particle statistics (both bosonic and fermionic) for quantum information processing [1,2]. In fact, it was shown that a useful task such as entanglement concentration could be accomplished, even if non-optimally, using only the effects of quantum statistics, without the need of any other interactions [2]. The above investigations differ significantly from some previous suggestions, where either anyonic statistics [3] or the effects of electronic statistics in conjunction with other interactions [4,5] are used for quantum information processing. Schemes using only particle statistics [1,2] would be very useful for tasks implemented with particles that interact very weakly or not at all with other identical particles, such as photons or neutrons. Such schemes are also extremely general in the sense of being independent of the actual particle species. It is however not known whether such schemes can accomplish quantum information processing *efficiently.*

Here we present a particular quantum information processing task involving two qubits encoded in the internal degrees of freedom of identical particles and show that it can be performed *optimally* using *only* the effects of particle statistics. More specifically, we present a strategy for discriminating between two non-orthogonal states of two qubits using 50/50 beam splitters. Our strategy varies for fermions and bosons, and is optimal in both cases. The beam splitters are capable of performing the perfect discrimination between symmetric and anti-symmetric states, and this feature can be used to measure the absolute value of the total spin of two spin-half particles (e.g., electrons) without the need for any controlled operation between the qubits themselves. Moreover, we point out how the task of discriminating quantum states can be applied to detecting entanglement and purifying mixed states. We also discuss how to generalize these tasks to N qubits using N-port beam splitters and argue that quantum statistics could be used to perform even this generalized task optimally. We calculate in addition the Helstrom probability of optimal discrimination [6] for the general case of N qubits. We show by explicit calculation that this probability can be achieved in a fermionic three-port beam splitter strategy and that it is the same as the fermionic strategy for general N if the fermions are considered as classical particles that obey the Pauli exclusion principle as the only additional constrain. An advantage of our discrimination scheme is that it can also be easily implemented with the current technology. While the two qubits and other small N qubits versions of our protocol can be tested with electrons, photons, neutrons, etc., the large number of qubits versions could have interesting implementations in optical lattices [7].

Our work suggests a number of interesting research directions. One problem is to prove the optimality of the beam splitter strategy in the case of N qubits and its application to multiparty entanglement detection. This, we hope, will answer the question of weather the symmetry between fermions and bosons in our strategy will be preserved for a generalized beam splitter. It may also lead to a simple and physically intuitive selection principle governing bosonic behavior. Another possible direction is to classify all the pairs of states that can be optimally discriminated with our scheme. Finally, our results raise the hope that particle statistics can make quantum computation even more efficient and that this is an idea very much worth exploring in the future.

The authors thank P. Castagnoli, F. A. Bovino and P. Varisco for useful discussions on the role of particle statistics in quantum information processing. S.B. acknowledges support from the NSF under Grant Number EIA-00860368. Y.O. acknowledges support from Fundação para a Ciência e a Tecnologia from Portugal and wishes to thank the Institute for Quantum Information at Caltech for their hospitality. N.P. and V.V. thank Elsag S.p.A. for financial support and Centro de Física de Plasmas at IST in Lisbon for their hospitality. V.V. acknowledges support from Hewlett-Packard company, EPSRC and the European Union projects EQUIP and TOPQIP. This work has been partly supported by the QUIPROCONE collaboration grant number 044.

[1] Y. Omar, N. Paunković, S. Bose and V. Vedral, Phys. Rev. A **65**, 062305 (2002).
[2] N. Paunković, Y. Omar, S. Bose and V. Vedral, Phys. Rev. Lett. **88**, 187903 (2002).
[3] A. Yu. Kitaev, quant-ph/9707021 (1997).
[4] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[5] A. T. Costa, Jr. and S. Bose, Phys. Rev. Lett. **87**, 277901 (2001).
[6] C. W. Helstrom, *Quantum Detection and Estimation Theory*, Academic Press, New York (1976).
[7] J. Pachos and P. L. Knight, quant-ph/0301084 (2003).

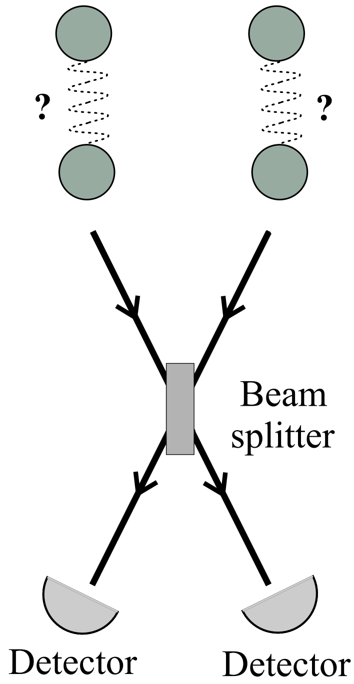

FIG. 1. This figure represents the set-up for our entanglement detection scheme. We have two pairs of identical particles, both of which are either in the same pure separable state or in the same pure maximally entangled state. We take one particle from each pair and interfere them at a beam splitter. If the states are disentangled then the particles are indistinguishable and, depending on the statistics, will either only bunch or only antibunch. Otherwise, if the states are maximally entangled, the particles are maximally mixed, meaning that they can be (probabilistically) distinguished and hence the statistics does not influence their behavior. This entanglement detection is a particular instance of our state discrimination scheme discussed in the paper.

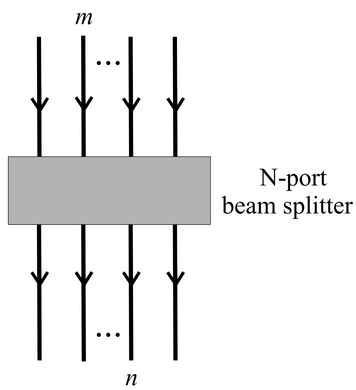

FIG. 2. This diagram represents a muliport beam splitter with N inputs and N outputs. The overall output state depends not only on the input, but also on the statistics (either fermionic or bosonic) of the identical particles involved. We have labelled two arbitrary ports, the m-th input port and the n-th output port.