Quantum Nonlinearities and Environmental Degrees of Freedom

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Nonlinearities have always played an important role in atomic physics and quantum optics. However, the size of these nonlinearities tends to be relatively small compared to the first-order (linear) processes. This is because of the weak nature of the coupling between atomic systems and environmental electromagnetic fields. In solid state devices the coupling between the solid state and the environmental fields tends to be stronger. The introduction of strong coupling between quantum devices and the environment can produce a range of nonlinear behaviour: e.g. chaotic-like behaviour in the quantum system or the nonlinear backreaction of the quantum device on the environment.

Although the strong coupling between the solid state device and the environment does provide interesting nonlinear behaviour, it also has a down side. In the absence of environmental fluctuations, a quantum mechanical device provides the physical basis for quantum information processing: quantum communications or quantum computing. A two-state qubit (or, more generally, a d-state system or ‘qudit’) can be used as the basis for a new generation powerful computer algorithms that offer significant reductions in processing time: Shor’s factoring algorithm and Grover’s database search algorithm are the two standard examples. Environmental fluctuations reduce the efficiency of these algorithms by interfering with the coherent evolution of the quantum device. Reducing the effect of environmental noise is crucial to the efficient operation of quantum processors. However, a completely isolated quantum system cannot be interrogated to find the result of a quantum computation, manipulated by means of external control fields, or reinitialised to start another calculation. Clearly some coupling is required to allow the quantum system to be interrogated and manipulated, but the coupling should not introduce too much decoherence or dissipation. A variety of methods for reducing the effect of environmental noise have been proposed, including duplicating processing resources to allow error correction, using time-dependent pulses to decouple the decoherent environment dynamically, and using a restricted quantum sub-space that only couples weakly to the environment for processing and a larger sub-space for interrogating and initialising the system.

In this paper, we examine the different ways in which the interaction between a quantum device and the electromagnetic modes representing an environment can be described and the different types of nonlinear behaviour that they can produce. We also examine the way in which strong environmental coupling, and the nonlinearities that it can produce, could affect the ability to control the environmental decoherence in quantum processors. In particular, we concentrate on similarities between the nonlinear behaviour found in complex classical systems and the description of nonlinear behaviour in distributed quantum systems using the time-dependent Schrödinger equation in the presence of noise and quantum trajectory theories (possibly representing the behaviour of individual quantum systems) and the ensemble-averaged behaviour of an open quantum system as described by the Master equation for the reduced density operator.