# Robust quantum computing with dark states and composite pulses

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Research in quantum computing (QC) is developing rapidly and there is an increasing need for being able to carry out quantum gate operations of high fidelity. In the present work it is demonstrated that high fidelity gate operations can be carried out in an optically addressed ensemble quantum computing system, where the individual qubit constituents can have a distribution of different oscillator strengths and different resonance frequencies.

We present our analysis with a particular proposal in mind, which makes use of ion impurities in crystals for quantum computing. Two low lying states in the ions may be used as qubit levels. These states are to a good approximation split by the same energy separation for all ions, whereas an optical transition to an excited state $|e\rangle$ experiences strong inhomogenoeus broadening. In the rare-earth-ion quantum computing (REQC) scheme [1], it is proposed to define qubits as the ions which have this optical transition frequency in certain frequency channels within the inhomogeneously broadened line of the ensemble. Ions in the vicinity of the desired frequency are removed by spectral hole burning techniques, see Fig. 1. Pairs of interacting qubits $(i, j)$ are subsquently identified spectroscopically, by pumping away those ions which do not have their transition frequency $\nu_{j(i)}$ shifted by the dipole-field, when neighboring ions in the channel $i(j)$ are excited at their frequency $\nu_{i(j)}$. Gates are employed by coupling the qubit states via the excited state, and two-bit gates are employed by use of the frequency shifts that one can apply to one ion conditioned on the state of the other one [1]. The implementation scheme proposed in Ref. [1] for a C-NOT operation in REQC, with qubit $i$ as control bit and qubit $j$ as target bit, comprise the following steps:

1. $\pi$-pulse on $|0\rangle_i - |e\rangle_i$
2. $\pi$-pulse on $|0\rangle_j - |e\rangle_j$
3. $\pi$-pulse on $|1\rangle_j - |e\rangle_j$
4. $\pi$-pulse on $|0\rangle_j - |e\rangle_j$
5. $\pi$-pulse on $|0\rangle_i - |e\rangle_i$

If qubit $i$ is initially in state $|0\rangle$, it is excited, and the frequency shift of qubit $j$ makes the steps 2,3 and 4 non-resonant, hence nothing happens, and ion $i$ is safely returned to its initial state. If qubit $i$ is in state $|1\rangle$, there is no frequency shift, and the resonant processes 2,3 and 4, effectively exchange states $|0\rangle_j$ and $|1\rangle_j$, as desired for the operation.

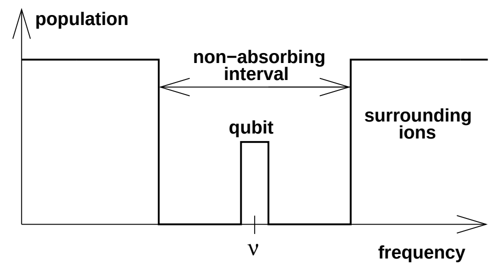


Figure 1: A schematic illustration of a qubit in an otherwise non-absorbing spectral interval. $\nu$ denotes the central absorption frequency of the channel.

To have a sufficiently large number of active ions, and hence an appreciable number of quantum registers with, e.g., 3 or more coupled qubits, it is preferable to allow ions with frequencies in a not too narrow interval around the channel frequencies $\nu_i$ as shown in the figure. This means, however, that the transitions 1-5 will not be resonant for all ions, and normally this would seriously compromise the gate fidelity in the system. One could apply very short pulses which are less sensitive to the resonance criterion, but they would have the unfortunate effect of exciting ions outside the windows around the frequency channels, and in the two-bit gates they would partially remove the sensitivity to the frequency shift on ion $j$ due to the excitation of ion $i$. Instead, we suggest to apply so called composite pulse sequences [2], which have already been used to battle systematic errors and experimental imperfections in NMR studies. $\pi$-pulses that transfer atoms between the upper and lower states of a transition, i.e. between the poles on the Bloch sphere, with high probability for a broad range of frequency detunings can be achieved by the composite pulse $90_{90}180_0 90_{90}$[1], where each rotation is formed by a square pulse of given duration and phase of the field. This pulse sequence is tolerant to frequency detunings, but it does not satisfactorily exclude excitation of the surrounding ions, because of the high frequency components of the square pulses, and we have studied composite pulses based on gaussian field envelopes, for which a variational calculation gives an optimal pulse of the form $92.50_{96.98}192.00_{6.86}92.42_{96.23}$. Complex hyperbolic secant pulses offer an alternative process with the desired properties as shown by Silver et al. [3].

These pulses are excellent for steps 1–2 and 4–5 in the REQC C-NOT scheme, since the initial and final states are explicitly known for all these processes, hence we do not need a perfect unitary rotation for all input states. The inital state before step 3, however, is an unknown superposition state and a so-called class $A$ composite pulse, i.e. a pulse that produces a compensated rotation for all initial conditions is required.

It is difficult to form such composite pulses, and we propose an alternative robust technique making use of the dark states of the system: If we turn on fields that simultaneously couple $|0\rangle_j$ and $|1\rangle_j$ to $|e\rangle_j$, there will effectively be two ground state superpositions, that will be respectively coupled and un-coupled to the excited state. The uncoupled component is unchanged, whereas the coupled component can be put with almost certainty into the excited state and back again, and by this process, it can receive a phase of $\pi$ if it makes the equivalent of a complete Rabi cycle on the Bloch sphere. If the two fields have the same strengths, this phase shift in the basis of coupled and non-coupled states is equivalent to the NOT-operation in the qubit basis. General rotations in the qubit basis are obtained by adjusting the phase on the coupled ground state superposition.

A number of other proposals exist for quantum computing where qubit addressing is made by the transition frequency, shifted for different qubits, e.g., in the ion trap by magnetic field gradient, microwave fields or strong laser fields, and we hence believe that the central idea of using composite pulses and selective coupling to coherent superposition states will not only be applicable to the REQC quantum computing proposal.

[1] $\theta_\varphi$ is used to denote a rotation through an angle $\theta$ about an axis in the $xy$-plane of the Bloch sphere at an angle $\varphi$ from the $x$-axis.

## References


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