# $d$-chotomic measurement for the non-locality test of a continuous-variable state

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In the recent research on the violation of local realism by two entangled $d$-dimensional systems, Kaszlikowski and coworkers [1] adopted the noise fraction of a state as an objective magnitude of the violation of local realism. Using the numerical search on the all constraints of non-negative marginal joint probabilities, they showed that the violation of local realism is stronger for two maximally entangled qudit ($d$ dimensional quantum system, $d > 2$) than for two qubits and the magnitude of violation increases with the dimension $d$. In their analysis, a family of observables for measuring the joint probabilities are restricted to unbiased multiports whose bases are described in terms of quantum Fourier transformations (QFT).

A new set of Bell's inequalities for a bipartite qudit were developed by Collins *et al.* [2] based on the constraints that the correlations exhibited by local variable theories must be satisfied. This version of Bell's inequality reproduce the previous results [1] the amount of noise which is resistant to the locality. This implies that the Collins *et al.*'s inequalities are useful for the direct comparison of the amount of non-locality between different dimensional systems. Shortly after that, larger violation or equivalently, stronger resistance to noise, was found for a non-maximally entangled state using Collins *et al.*'s inequality by Acin *et al.* [3]. In their work, they constructed the Collins *et al.*'s Bell operator for the two three-level quantum systems using the QFT basis measurement and found the largest eigenvalue whose eigenvactor is confirmed to be a partially entangled state. They claimed that resistance to noise is not a good measure for non-locality. However, in the above discussions of the noise resistant non-locality test, it is still questionable whether the QFT is the most sufficient unitary transformation for the measurement on an arbitrary $d$ dimensional state. If that is not the case then the conclusions driven by above works should be modified and a more general statement could be given.

The potentiality of an infinite dimensional system as a resource for quantum information processing goes intuitively beyond that of a qubit system. Testing of quantum nonlocality for the continuous-variable (CV) system has been studied in various context. Banaszek and Wódkiewicz [4] proposed an experimentally realizable scheme for testing non-locality of a correlated two-mode quantum state of light. In the scheme, the observables which vary in terms of the displacement operators lead to the phase space measurement that is described by the qusiprobability $Q$ function or the Wigner function. Their scheme allows us to use the CH and CHSH version of Bell's inequalities for the non-locality test since the observables are attributed by dichotomic outcomes, either of the photon presence or of the parity of the registered number of photons. By introducing a different measurement on a CV state in an extended way of the usual spin-1/2 system, Chen *et al.* showed the EPR state could result in the maximal violation of CHSH Bell's inequality [5]. They introduced the "pseudo spin" operators for photons, which are defined as the direct tensor summation of an infinite number of pauli spin matrices. Their observables may be interpreted as an extension of Gisin-Peres observables to an infinite-dimensional system [6]. Most recently, as a new approach on the CV state, Brukner *et al.* [7] established a correspondence between a continuous variable quantum system and a discrete quantum system of arbitrary dimensions.

In this work, we introduce a *$d$ modulo projection measurement* for the purpose of non-locality test on a CV system. The $d$ modulo projection measurement allows a finite $d$ number of outcomes for a CV system and it enables us to use the Collins *et al.*'s version of Bell's inequality which was originally suggested for the non-locality test of a qudit system. The $d$ modulo projection measurement is defined as the combinations of a $d$ modulo projector and the most general observable which varies by SU($d$) group transformations. Note that the "pseudo spin" observable in the Chen's formalism [5] is the specific case of our approach.

Fig. 1 shows the violation of Collins et al.'s version of Bell's inequality in our approach for the two-mode squeezed state as the squeezing parameter $r$ increases. It further exhibits comparisons among the different outcome measurements: dichotomic, trichotomic, qudrachotomic and quintchotomic measurements. Here, we use the numerical optimization in SU($d$) space which has the overall $4(d^2 - 1)$ parameters necessary to be adjusted to find the maximal violation. Employing the SU($d$) transformation implies that the most general measurement set is considered for the case of $d$ outcome measurement. As a results, the two-mode squeezed state always violate the Bell's inequality except the $r = 0$, regardless of the number of measurement outcome. Interestingly, although the amount of violation increases for the larger squeezing parameter, the infinite squeezing does not necessarily give the maximal violation of the Bell's inequality. At the infinite squeezing limit, the two-mode squeezed state becomes the EPR state which is projected onto the maximally entangle finite dimensional state. The amount of violation that we obtain for the EPR state agrees with the previous works [1,2] in which they only considered the QFT unitary transformation for the measurement rotation. This implies that for the case of maximally entangled state, QFT is a sufficient unitary transformation for the maximally attainable violation of the Bell's inequality. In the region of partially entangled state (finite values of squeezing parameter), the stronger violation than maximally entangled state is found and this values are even higher than the maximal violation found by Acin *et al.* [3]. This implies QFT is not the sufficient transformation for the Collins *et al.* version of Bell's inequality when the state is in a partially entangled state. In fact, except for the two modulo measurement, there is always the region where less entangled state gives more violation of the Bell's inequality as for a finite squeezing parameter. The maximum of the violation is monotonically increased as the number of outcome becomes higher.

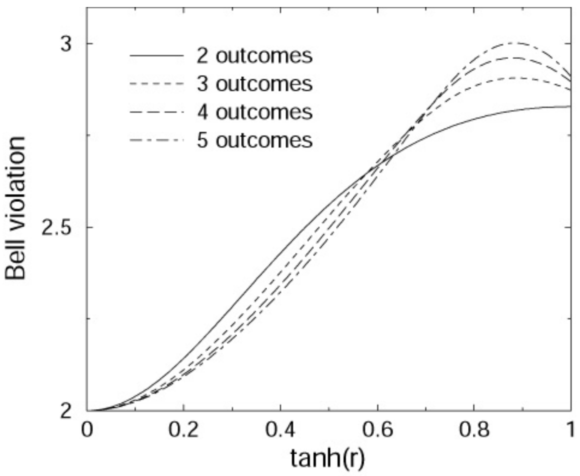


FIG. 1. Violation of the Collins *et al.* version of Bell's inequality for two-mode squeezed state with a finite number of measurement outcome.

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