Implementing quantum noiseless coding using linear optics

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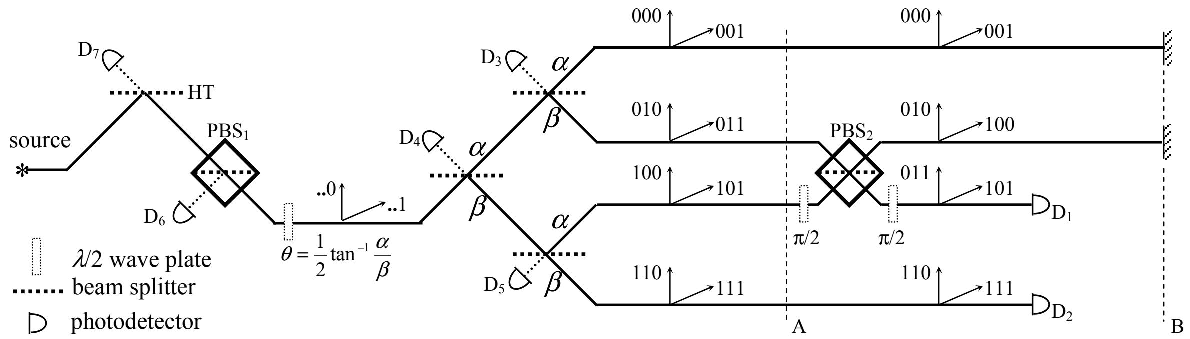
The coding of messages is a fundamental issue in information theory. Source coding entails coding common sequences of messages as short sequences of code letters, and uncommon sequences as long sequences. Recently Schumacher derived the quantum version of the source coding theorem [1]. It is based on reducing the redundancy that occurs when the letter states are non-orthogonal. For example, consider a message comprising the non-orthogonal letter states $|\psi_a\rangle$, $|\psi_b\rangle$, $|\psi_c\rangle$, … with corresponding probabilities P_a, P_b, P_c, … The quantum noiseless coding theorem states that by coding the quantum message into blocks of K letters, $KS(\rho)$ qubits are necessary to encode each block in the limit $K\to\infty$, where $S(\rho)$ is the von Neumann entropy of the density operator $\rho = \Sigma P_n|\psi_n\rangle\langle\psi_n|$.

We use the example given by Jozsa and Schumacher [2] of an alphabet consisting of two, equally-likely letters $|\psi_\pm\rangle$ in blocks of 3 letters, i.e. K=3. Here $|\psi_\pm\rangle = \alpha|0\rangle + \beta_\pm|1\rangle$ where $|0\rangle$ and $|1\rangle$ are an orthogonal computational basis, $\beta_\pm = \pm\beta$ and $\alpha^2+\beta^2=1$. The state of an arbitrary block $|B\rangle$ is given by

$$|B\rangle = |\psi_i\rangle\otimes|\psi_j\rangle\otimes|\psi_k\rangle = \alpha^3|000\rangle + \alpha^2(\beta_i|100\rangle+\beta_j|010\rangle+\beta_k|001\rangle) + \alpha(\beta_i\beta_j\,|110\rangle+\beta_j\beta_k\,|011\rangle+\beta_k\beta_i|101\rangle) + \beta_i\beta_j\beta_k|111\rangle$$

where i, j, and $k \in \{+, -\}$. The set of possible block states span an eight dimensional state space H. The compression entails first transforming with the unitary operator U, which maps two basis states as follows $U|100\rangle = |011\rangle$, $U|011\rangle = |100\rangle$ leaving the remaining computational basis states unchanged, and then projecting onto the state space Λ or its complement $H-\Lambda$. Here the subspace Λ is the four dimensional state space spanned by the states $|000\rangle$, $|011\rangle$, $|010\rangle$ and $|001\rangle$. The likelihood of projecting onto $H-\Lambda$ is proportional to β which can be made very small.

The figure shows a linear optical implementation of this scheme.



The message block is encoded using a single photon to represent one polarization and two path qubits. The encoding takes place to the left of the line marked A. The polarization qubit is encoded at the first $\lambda/2$ wave plate and the path qubits are encoded by the various beam splitters. The horizontal and vertical polarization modes (oblique and vertical arrows, respectively) for each path are labelled by binary digits according to the basis states they represent. The circuit shows the coding of all 3 qubits in the state $|\psi_+\rangle$; the state $|\psi_-\rangle$ of any qubit is generated by introducing π phase shifts in appropriate paths. In this idealised scheme, beam splitters with specific transmission and reflection coefficients (α^2, β^2) are used to define the path qubits. In practice the experiment employs combinations of polarizing beam splitters and wave plates to adjust the amplitudes along various paths. The unitary operation U is implemented by the polarizing beam splitter PBS_2 , and the projection onto the 'unlikely' subspace $H-\Lambda$ is realized by the detection of a photon by photodetectors D_1 and D_2. The two optical paths and two polarization modes at B represent the message block coded using just 2 qubits. In our experiment we decode the message by retroreflecting the light at the line B. The beam splitter marked HT has a high transmission coefficient. A retroreflected photon arriving at detector D_7 indicates perfect fidelity of the coding and decoding circuits. The actual average fidelity is determined by calculating the ratio of the number of photons detected by photodetector D_7 and other photodetectors.

We consider three different protocols corresponding to three different actions taken when the block state is projected into the 'unlikely' subspace $H-\Lambda$, and we present the results of recent experimental work.

[1] B. Schumacher, Phys. Rev. A **51**, 2738 (1995).
[2] R. Jozsa and B. Schumacher, J. Mod. Opts. 41, 2343 (1994).