



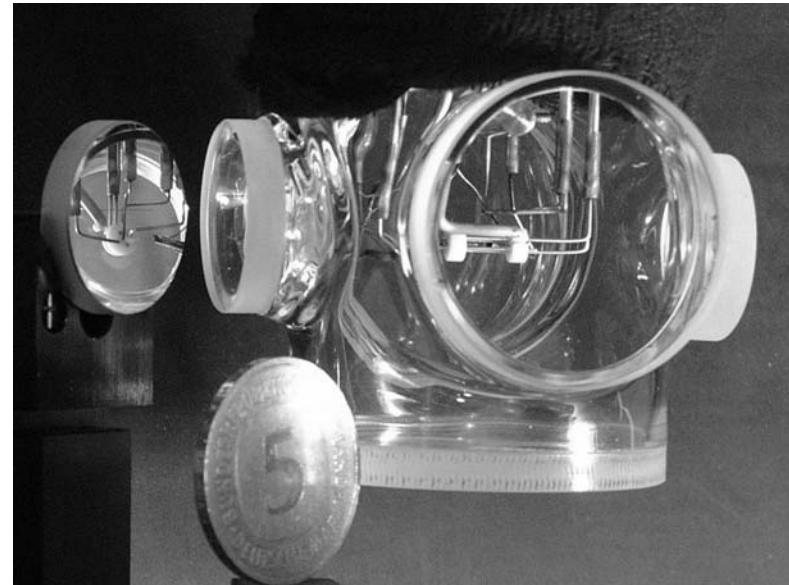
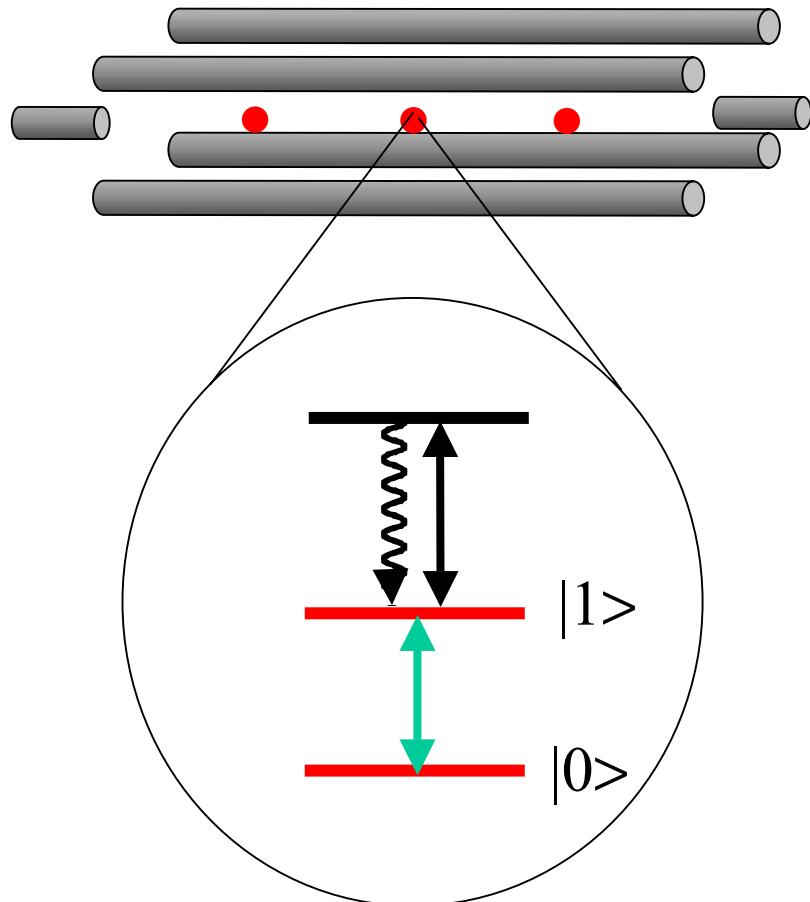
Universität Hamburg, Institut für Laser-Physik

Spin resonance with trapped ions: experiments and new concepts

Christof Wunderlich

Quantum information processing with trapped ions

Qubits



Internal states: qubits

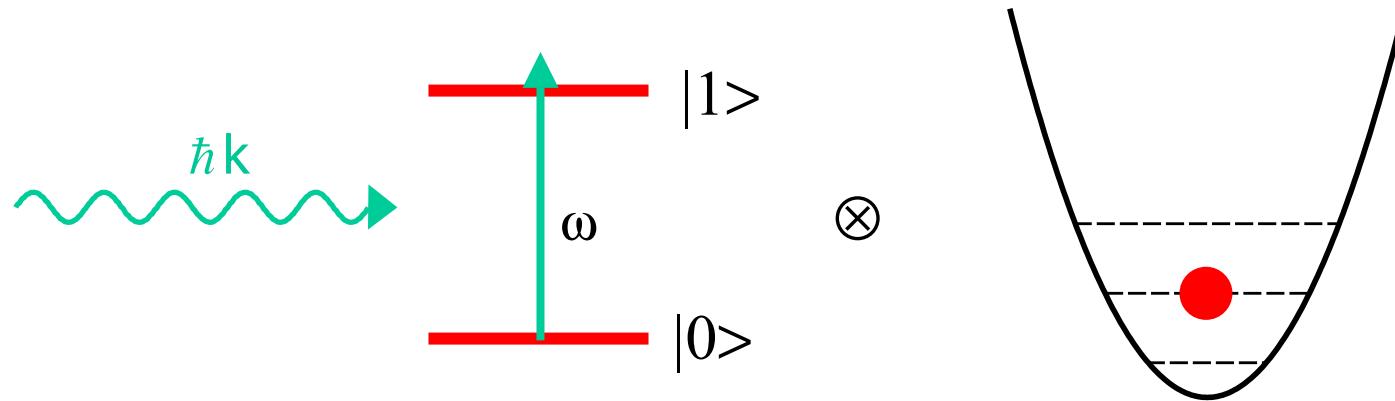
Vibrational motion: bus-qubit

J. I. Cirac, P. Zoller, PRL **74**, 4091 (1995)

QIP with trapped ions

Qubits and bus-qubit

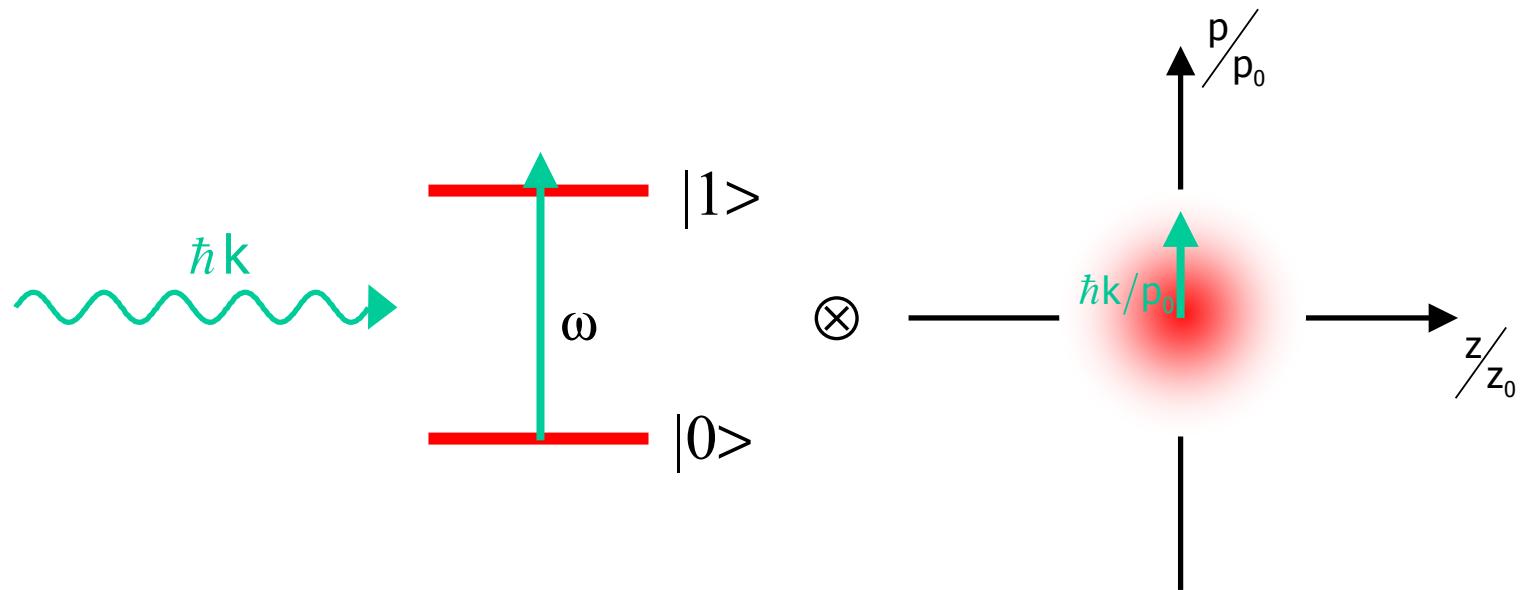
- Coupling internal and external dynamics:



QIP with trapped ions

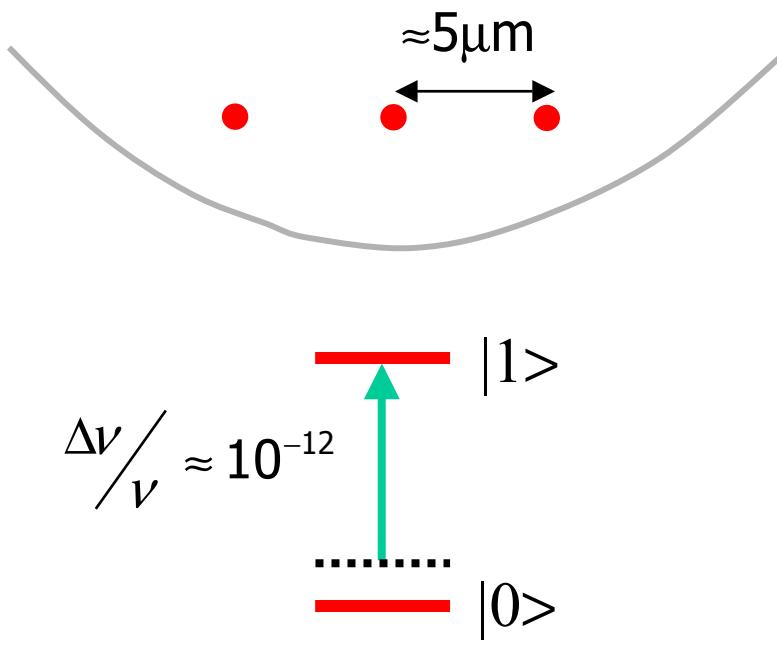
Qubits and bus-qubit

- Coupling internal and external dynamics:



$$H_I \propto \sigma_+ \exp[i\eta(a + a^\dagger)] + \text{h.c.} \quad \text{where} \quad \eta \equiv \frac{\hbar k}{2p_0} = \frac{z_0}{\lambda} 2\pi$$

QIP with trapped ions



Electromagnetic radiation used to

- **couple** internal and external degrees of freedom

$$\eta = \frac{z_0}{\lambda} 2\pi \quad z_0 \approx 10\text{nm}$$

\Rightarrow optical wavelengths

- **address** individual qubits

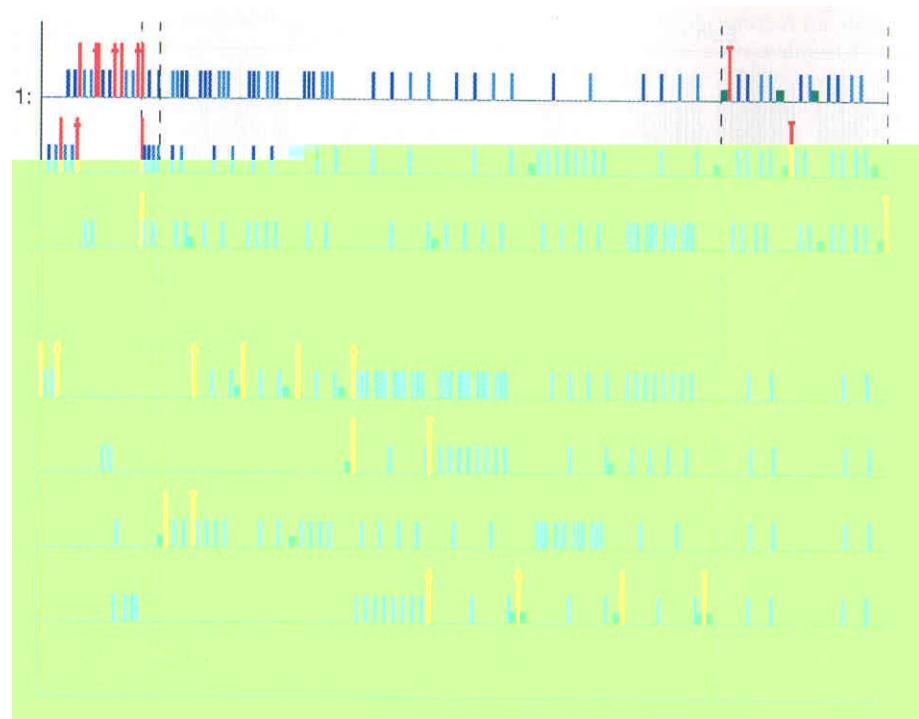
\Rightarrow optical wavelengths

\Rightarrow **Precise coherent operations demand:**

Small emission bandwidth, high absolute stability of frequency and intensity. Beam quality, pointing stability, diffraction.

Spin resonance

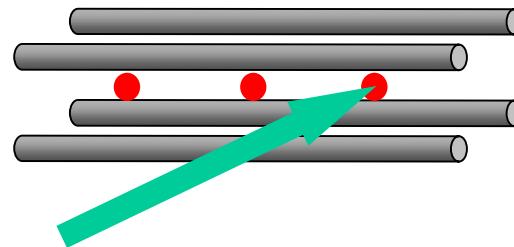
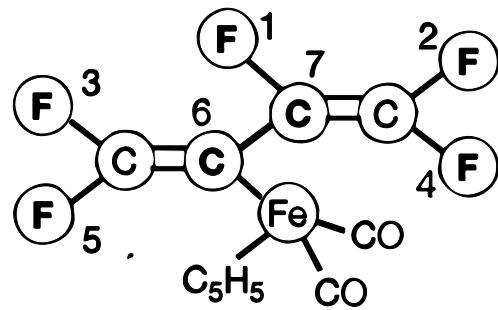
- Quantum algorithms demonstrated.
 - Sophisticated techniques.
 - **Technological basis:** coherent manipulation using rf and microwave radiation.
-
- Macroscopic ensemble
⇒ exponential cost
 - Design of molecules nontrivial



Vandersypen et al., Nature **414**, 883 (2001)

Spin resonance with trapped ions?

Ion traps:



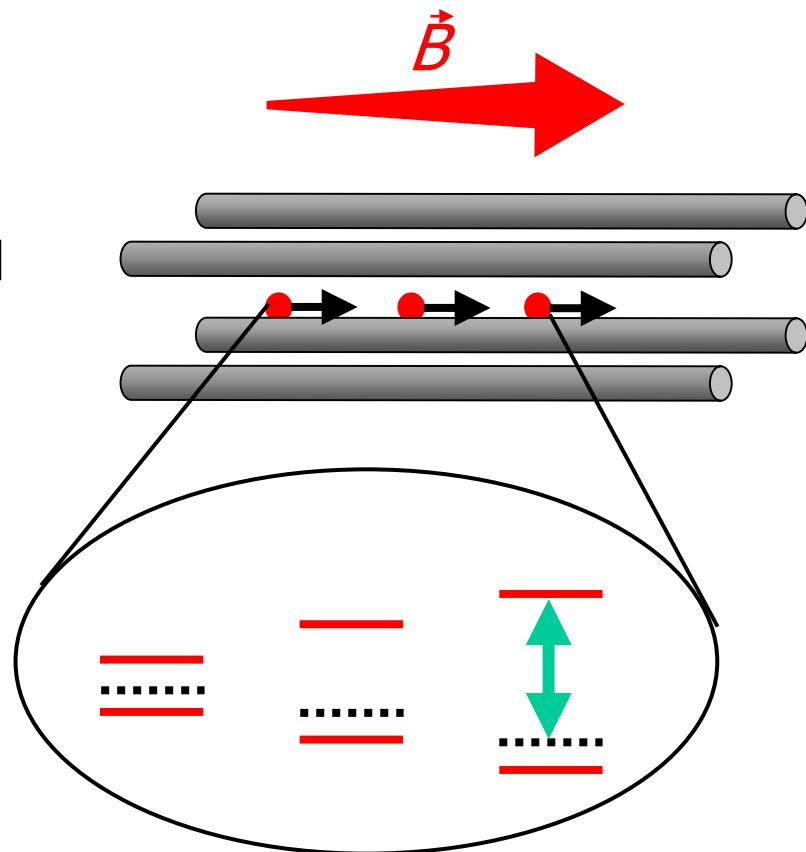
Macroscopic ensemble
⇒ exponential cost.
Design of molecules nontrivial.

Individual qubits.
Efficient readout.
Use microwaves?

Spin resonance with trapped ions

New concept:

- Qubit resonances shifted individually
- **Coupling** of internal and external dynamics

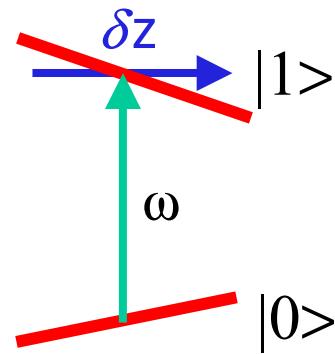


F. Mintert, Ch. Wunderlich,
Phys. Rev. Lett. **87**, 257904 (2001).

Spin resonance with trapped ions

- Coupling internal and external dynamics:

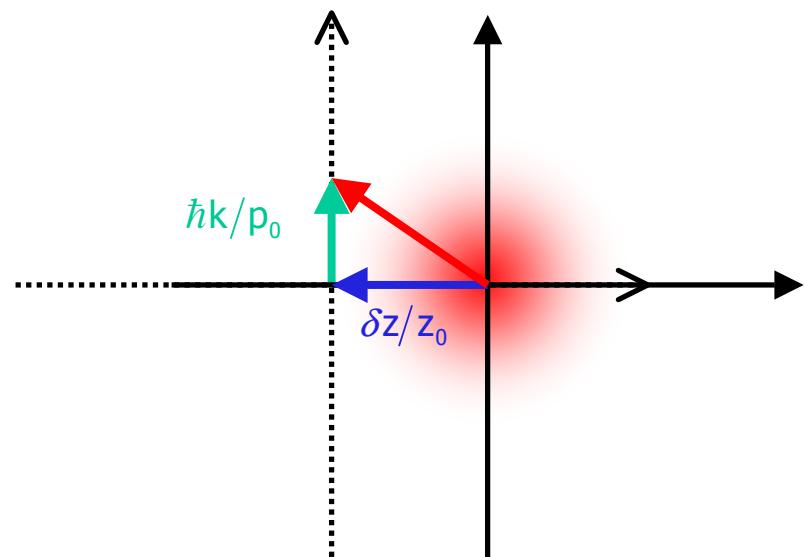
Magnetic field gradient



Equilibrium shifted by $\delta z = -\hbar \frac{\partial_z \omega}{m \nu^2}$

effective Lamb-Dicke parameter:

$$\eta' \equiv \eta - i\varepsilon$$

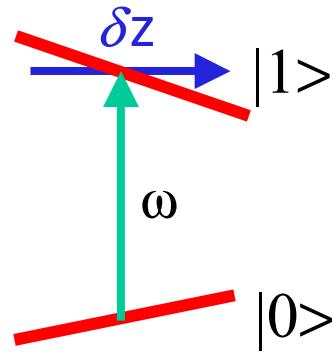


$$\text{where } \varepsilon \equiv \frac{\delta z}{z_0} = z_0 \frac{\partial_z \omega}{\nu}$$

Spin resonance with trapped ions

- Coupling internal and external dynamics:

Magnetic field gradient

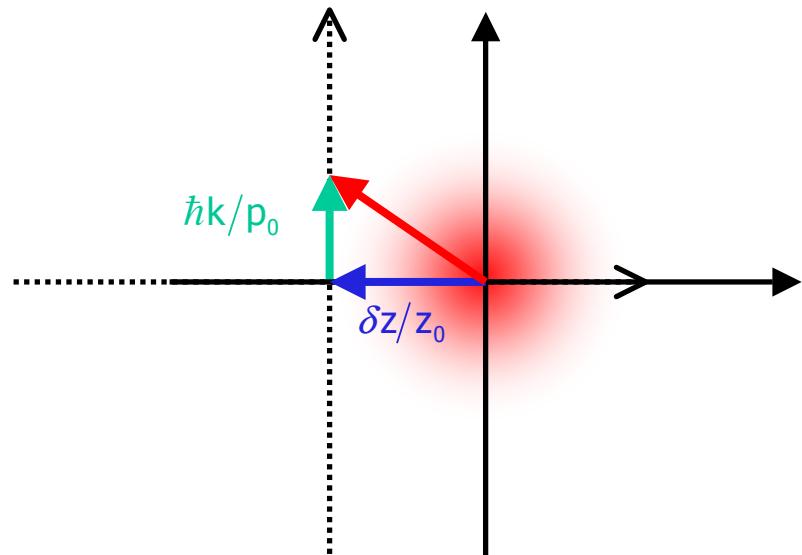


Equilibrium shifted by $\delta z = -\hbar \frac{\partial_z \omega}{m \nu^2}$

$H_I \propto \sigma_+ \exp[i\eta' (a + a^\dagger)] + \text{h.c.}$ where $\eta' \equiv (\eta^2 + \varepsilon^2)^{1/2}$

⇒ All optical schemes suitable for spin resonance

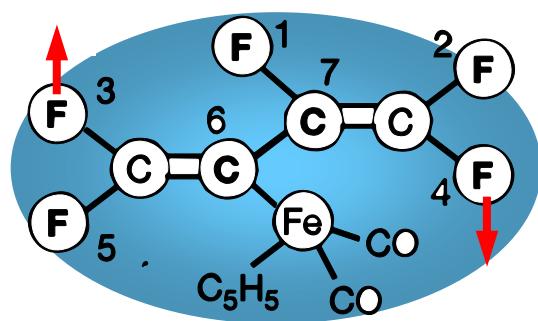
F. Mintert, Ch. Wunderlich, Phys. Rev. Lett. **87**, 257904 (2001).



Spin resonance with trapped ions

Analogy with NMR

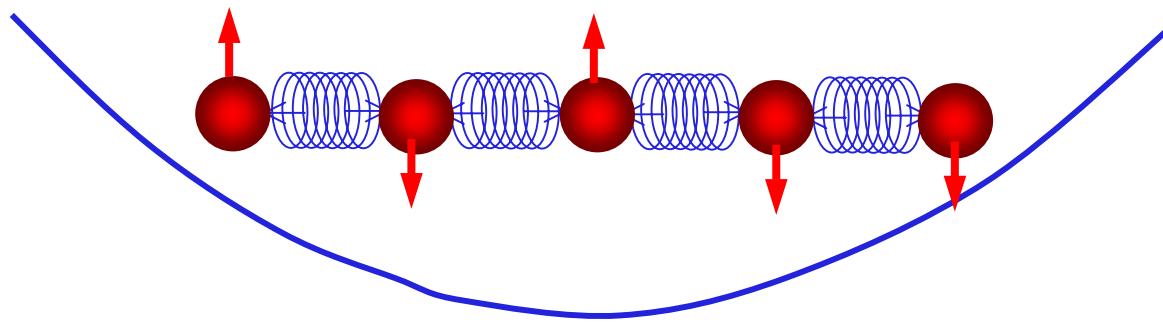
Conditional dynamics: $\hbar \sum_{n < l}^N \sigma_{z,n} \sigma_{z,l} J_{nl}$



Spin resonance with trapped ions

Analogy with NMR

Conditional dynamics: $\hbar \sum_{n<1}^N \sigma_{z,n} \sigma_{z,l} J_{nl}$ with $J_{nl} \propto \left(\frac{\partial_z B}{v_1} \right)^2$

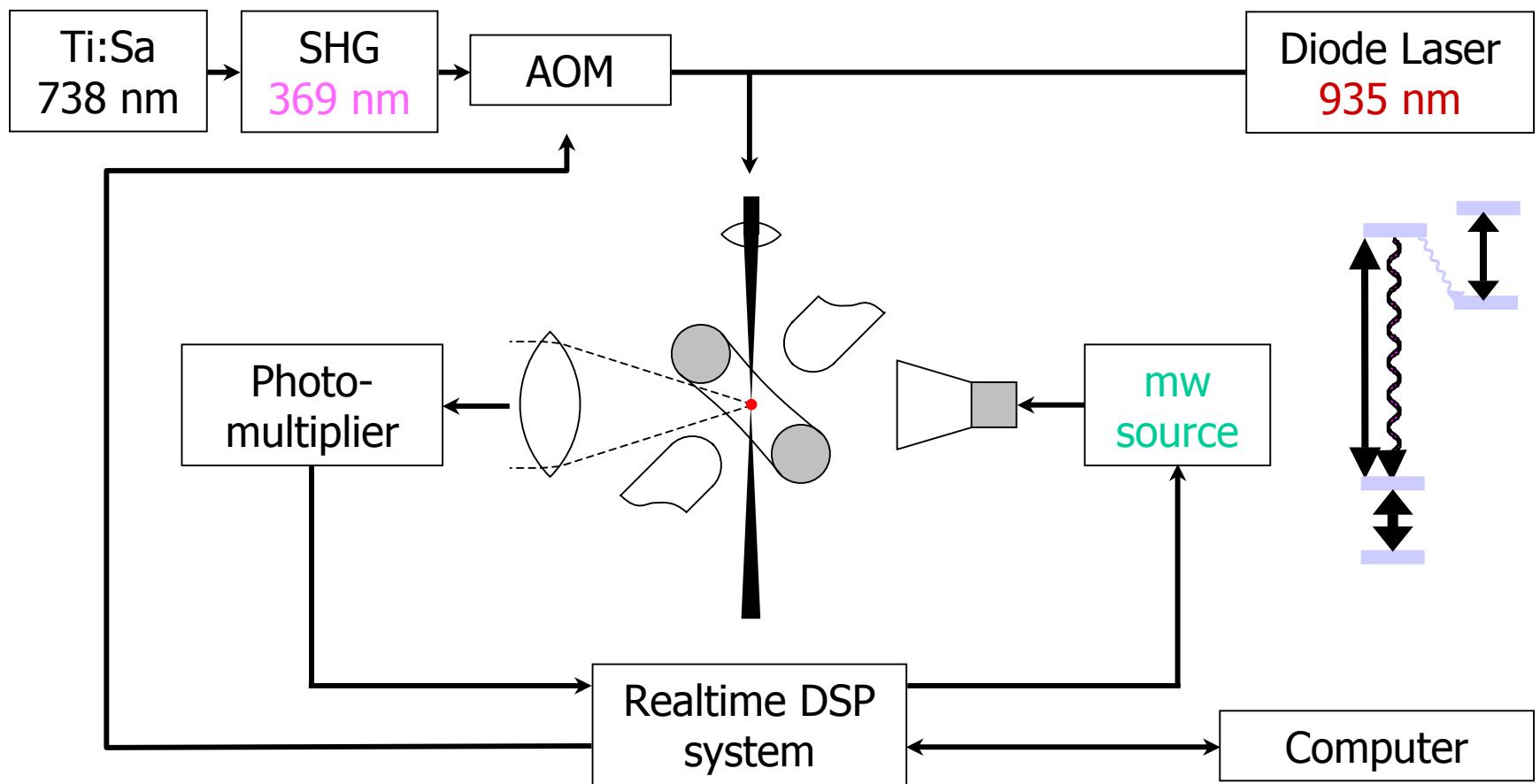


Single N-qubit „designer“ molecule:

- adjustable coupling constants
- Efficient read out of individual qubits

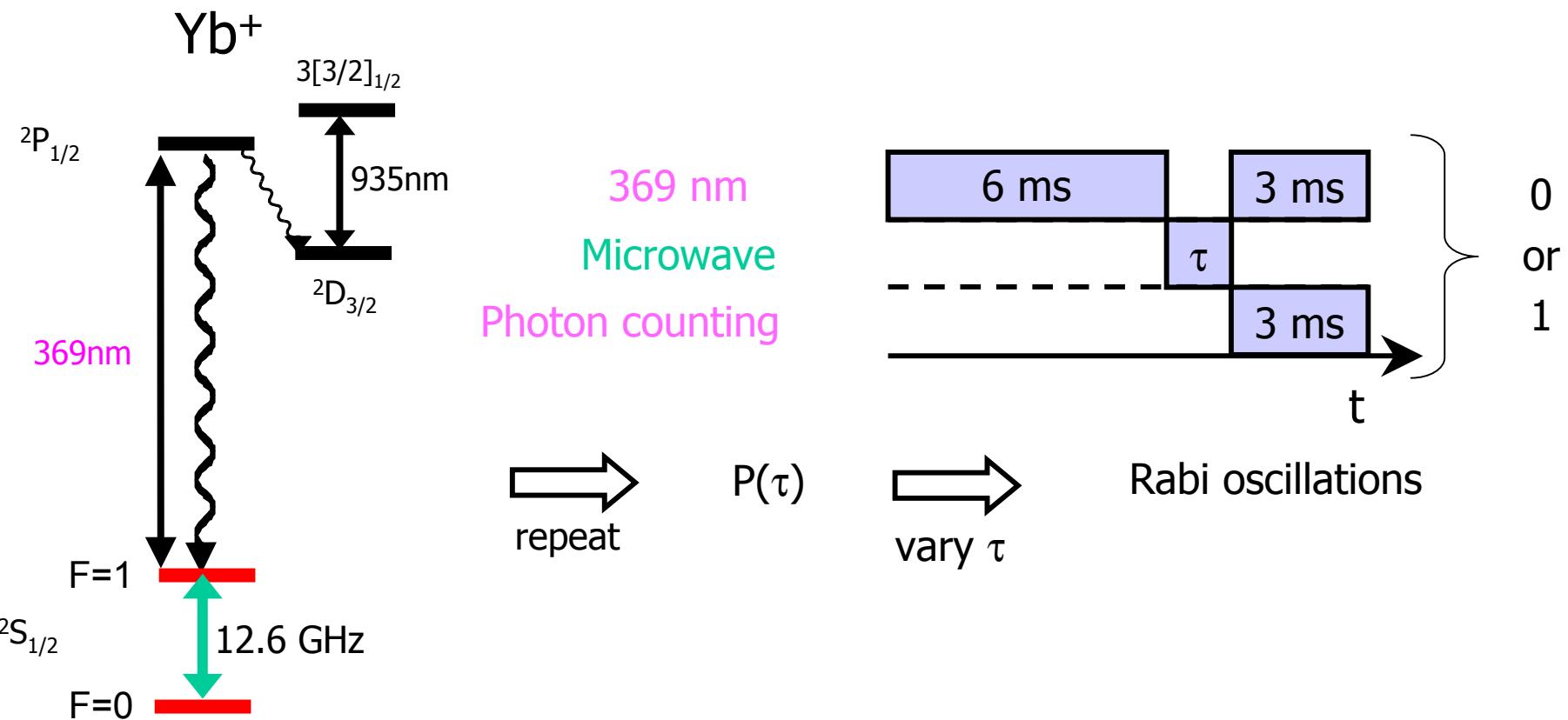
CW in *Laser Physics at the Limit*, Springer, 2002, p. 261.

Experiment Setup



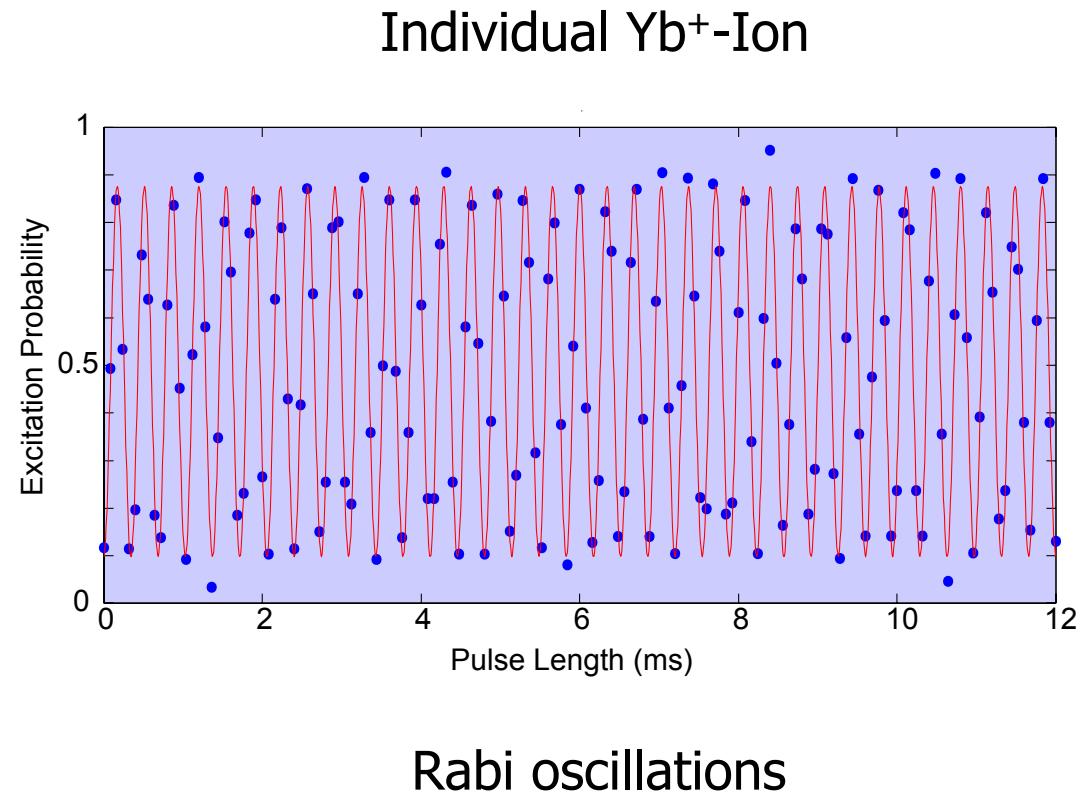
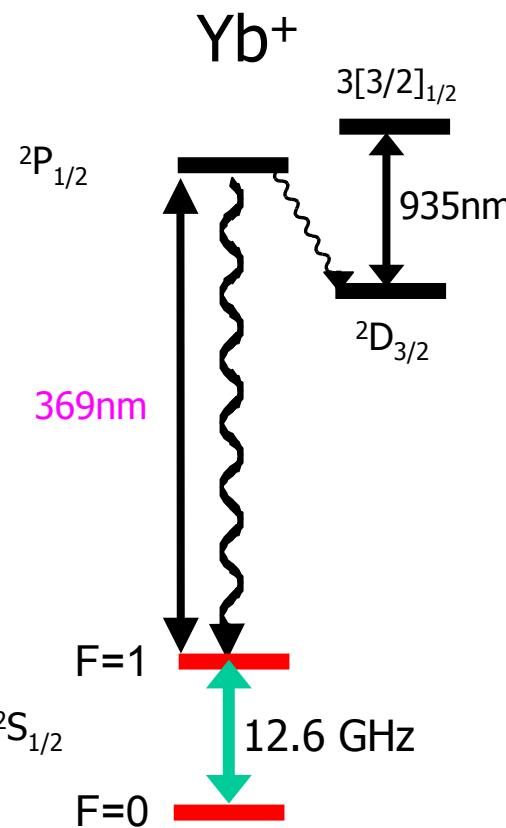
Trapped Yb⁺ ions

Coherent preparation of individual quantum systems



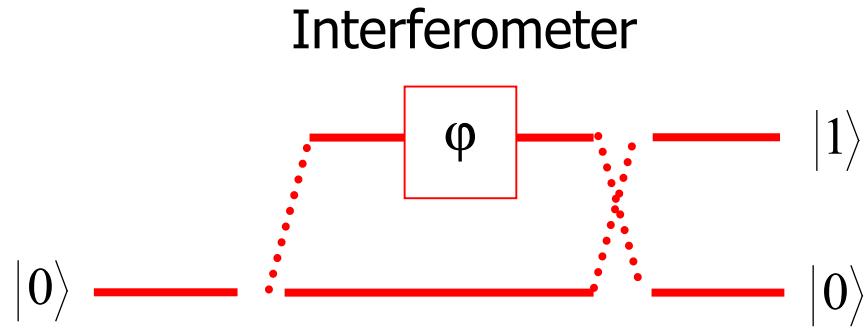
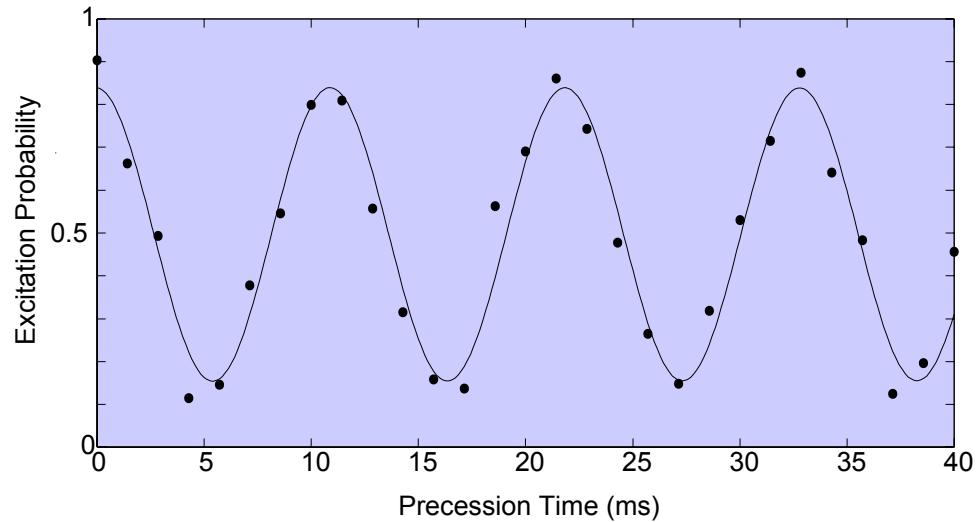
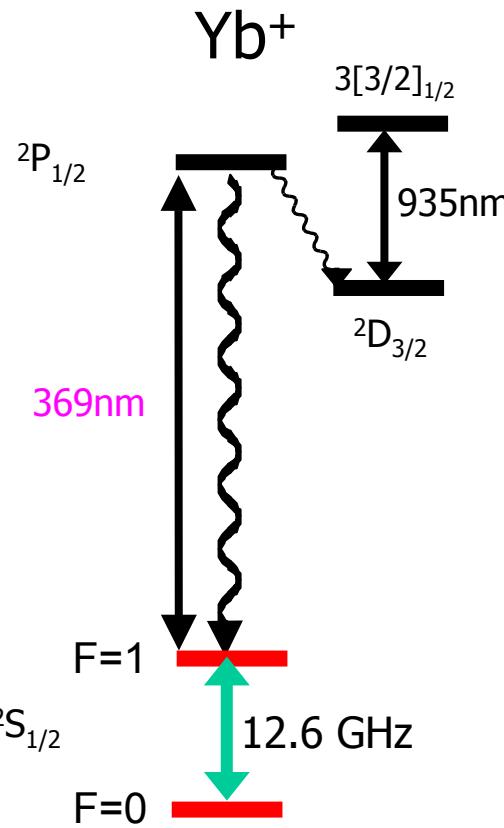
Trapped Yb⁺ ions

Coherent preparation of individual quantum systems

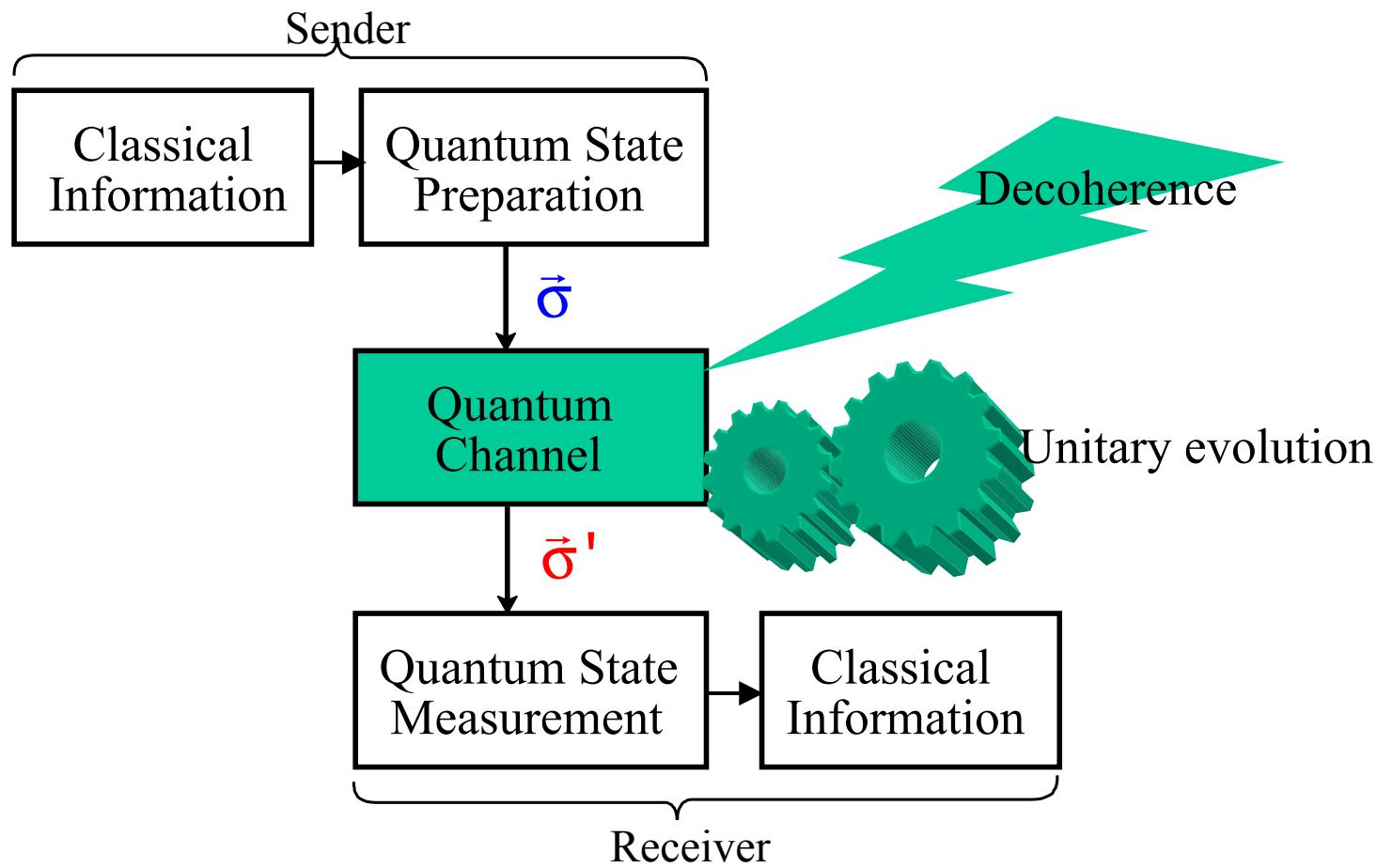


Trapped Yb⁺ ions

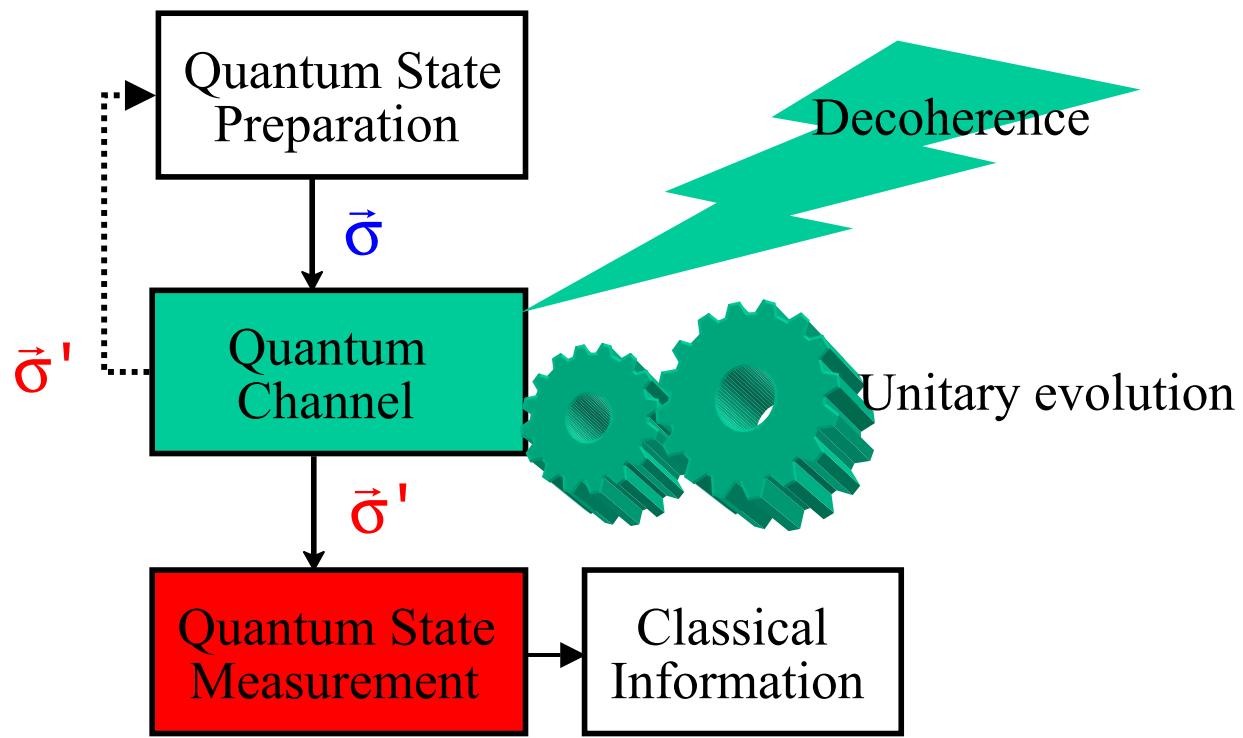
Coherent preparation of individual quantum systems



Qubit dynamics

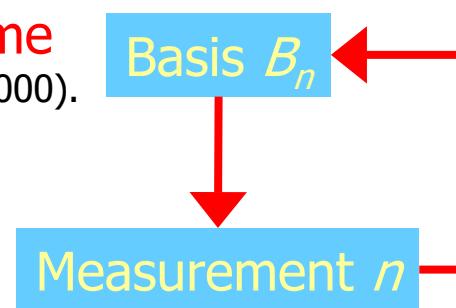


Qubit dynamics



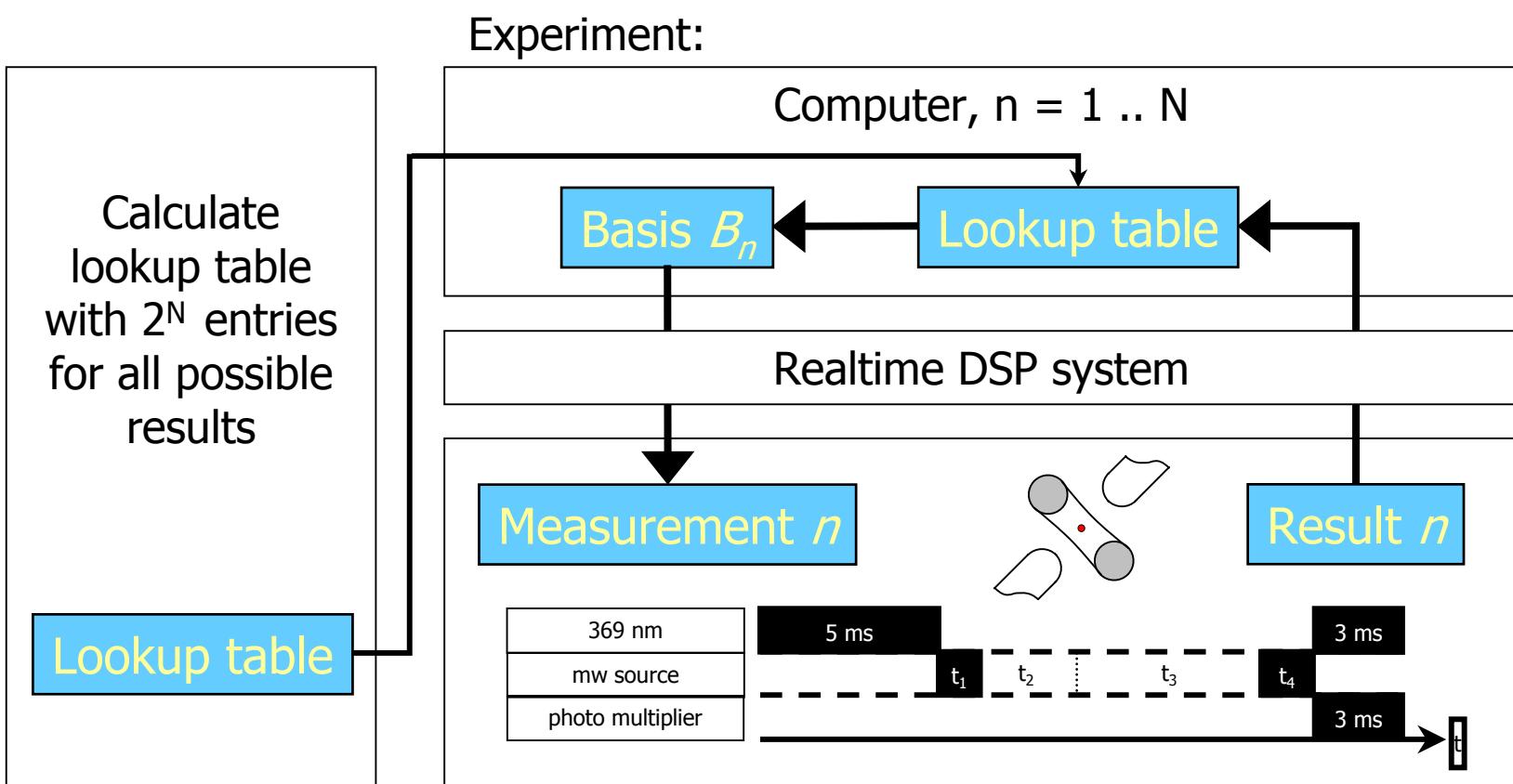
Determination of a quantum state

- Tomography: e.g. light fields, motional wave packets
M. Freyberger, P. Bardroff, C. Leichtle, G. Schrade, W. Schleich, Phys. World **10** No.11, 41 (1997).
- Estimation using a **finite** number N of identically prepared qubits:
 - Optimal estimation requires **entangled** basis
 $N=2$: A. Peres, W.K. Wootters, PRL **66**, 1119 (1991); S. Massar, S. Popescu, PRL **74**, 1259 (1995). $N \leq 5$: J. I. Latorre, P. Pascual, and R. Tarrach, PRL **81**, 1351 (1998).
 - First experiments ($N=2$)
V. Meyer et al., PRL **86**, 5870 (2001).
- **Separate** measurements on N qubits: **Adaptive scheme**
D. G. Fischer, S. H. Kienle, and M. Freyberger, Phys. Rev. A **61**, 032306 (2000).



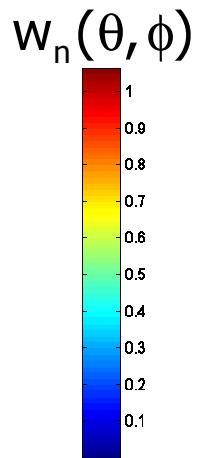
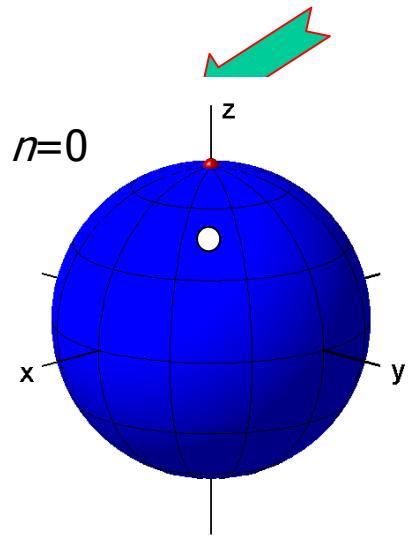
Experiment

Preparation and measurement



Estimating a quantum state

Experiment: self-learning estimation



- Probability density on Bloch sphere after measurement n .

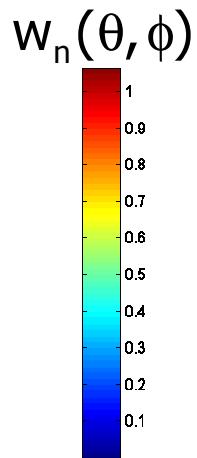
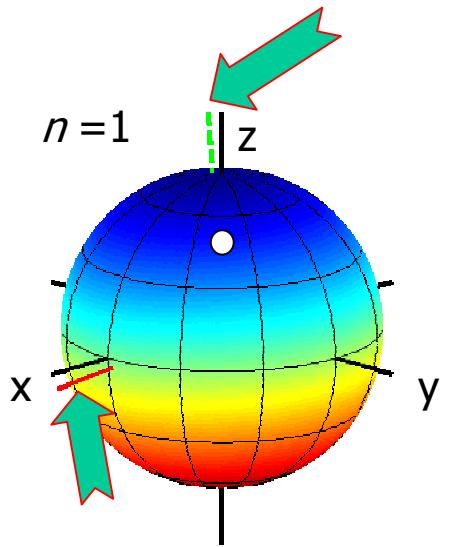
$$\rho_n = \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi w_n(\theta, \phi) |\theta, \phi\rangle \langle \theta, \phi|$$

- Calculate direction of next ($n+1$) measurement from $w_n(\theta, \phi)$.
Maximize classical information gain



Estimating a quantum state

Experiment: self-learning estimation



- Probability density on Bloch sphere after measurement n .

$$\rho_n = \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi w_n(\theta, \phi) |\theta, \phi\rangle \langle \theta, \phi|$$

- Calculate direction of next ($n+1$) measurement from $w_n(\theta, \phi)$.

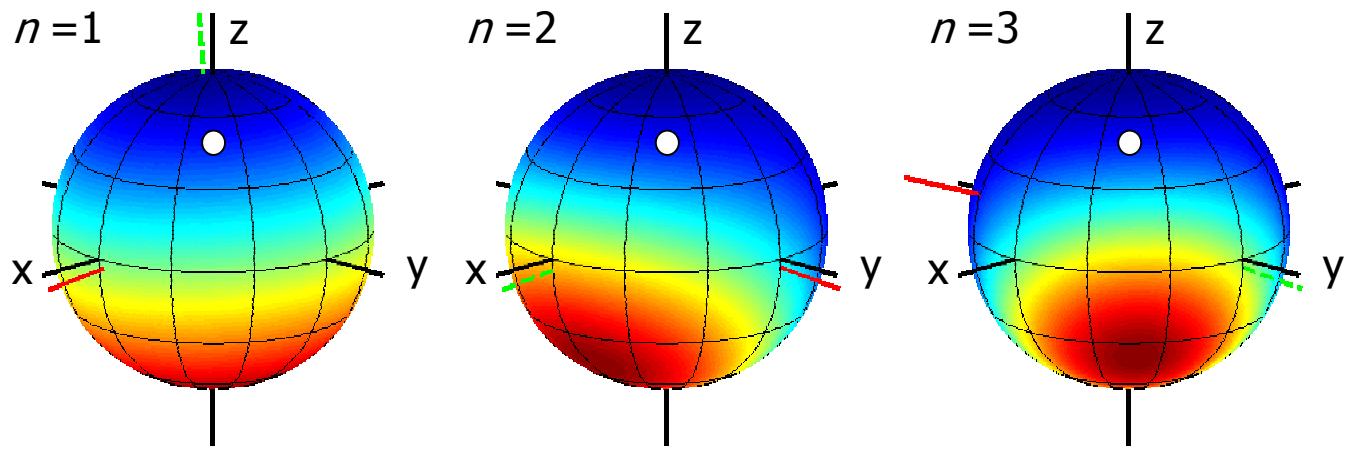
Better: Maximize expected Fidelity

$$F_{n+1}(\theta, \phi) = \langle \theta, \phi | \rho_{n+1} | \theta, \phi \rangle$$

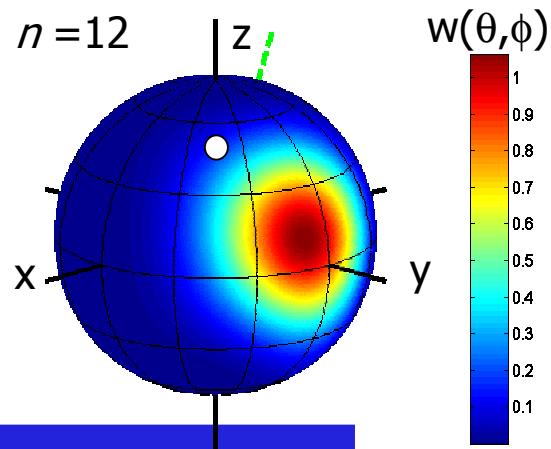


Estimating a quantum state

Experiment: self-learning estimation



■ ■ ■



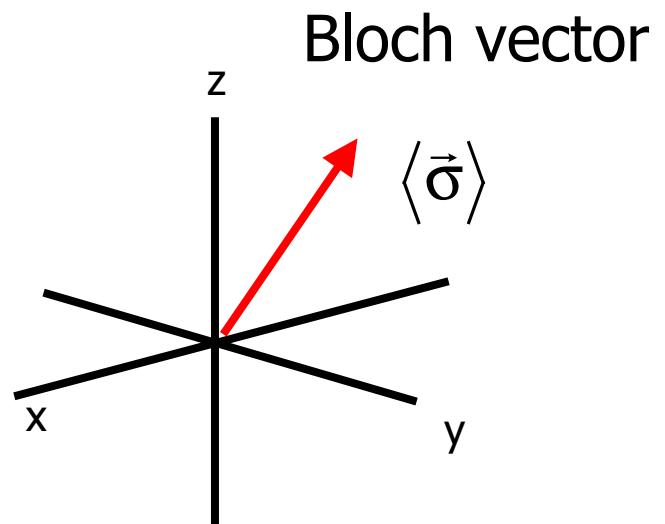
Th. Hannemann et al. PRA **65**, 050303(R) (2002)

Estimating a quantum state

Comparison with theory

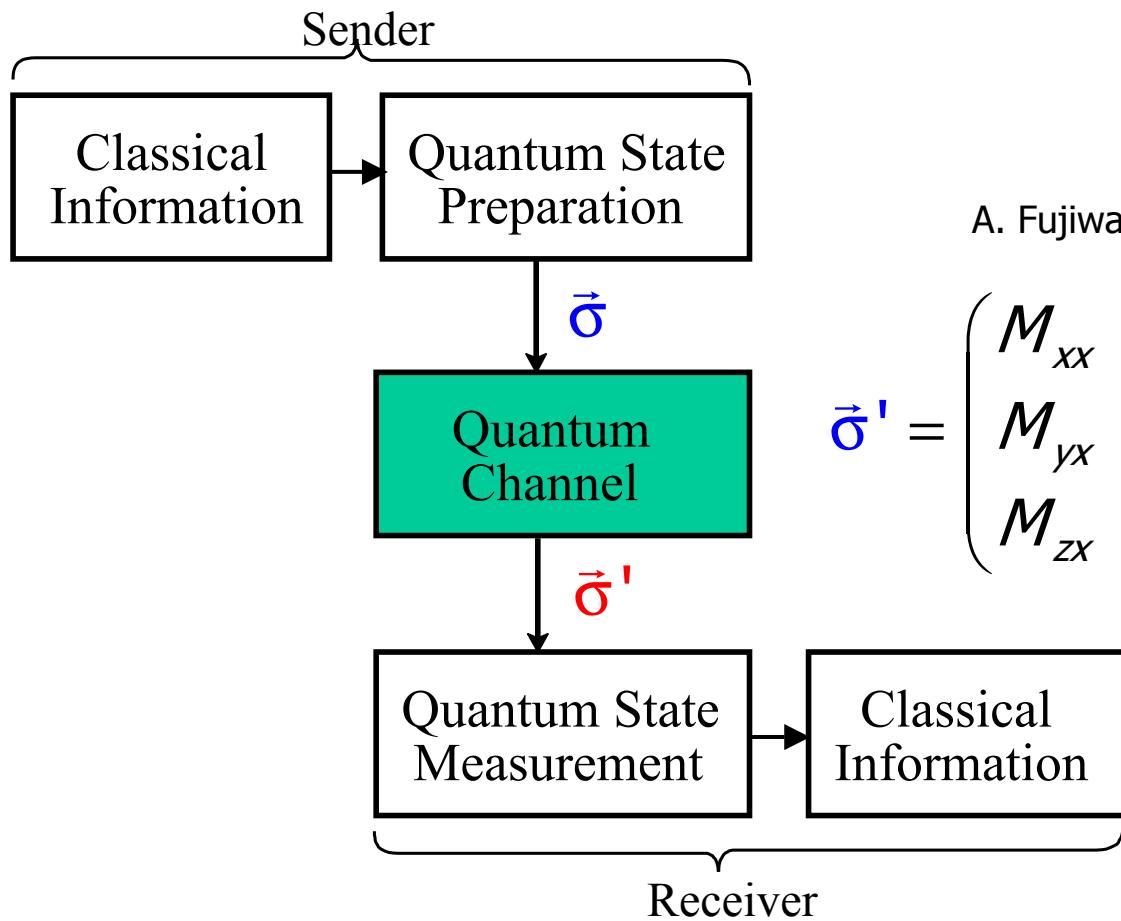
- Account for decoherence:

- Preparation $\eta_{\text{prep}} = 89 \%$
- Detection $\eta_{\text{det}} = 97 \%$



$\langle F \rangle_{\text{Exp}}$	$ \langle \vec{\sigma} \rangle $	$\langle F \rangle_{\text{Theo}}(\langle \vec{\sigma} \rangle)$
$(85.0 \pm 0.6) \%$	$(74.8 \pm 2.1) \%$	$(85.4 \pm 0.7) \%$

Realization of quantum channels

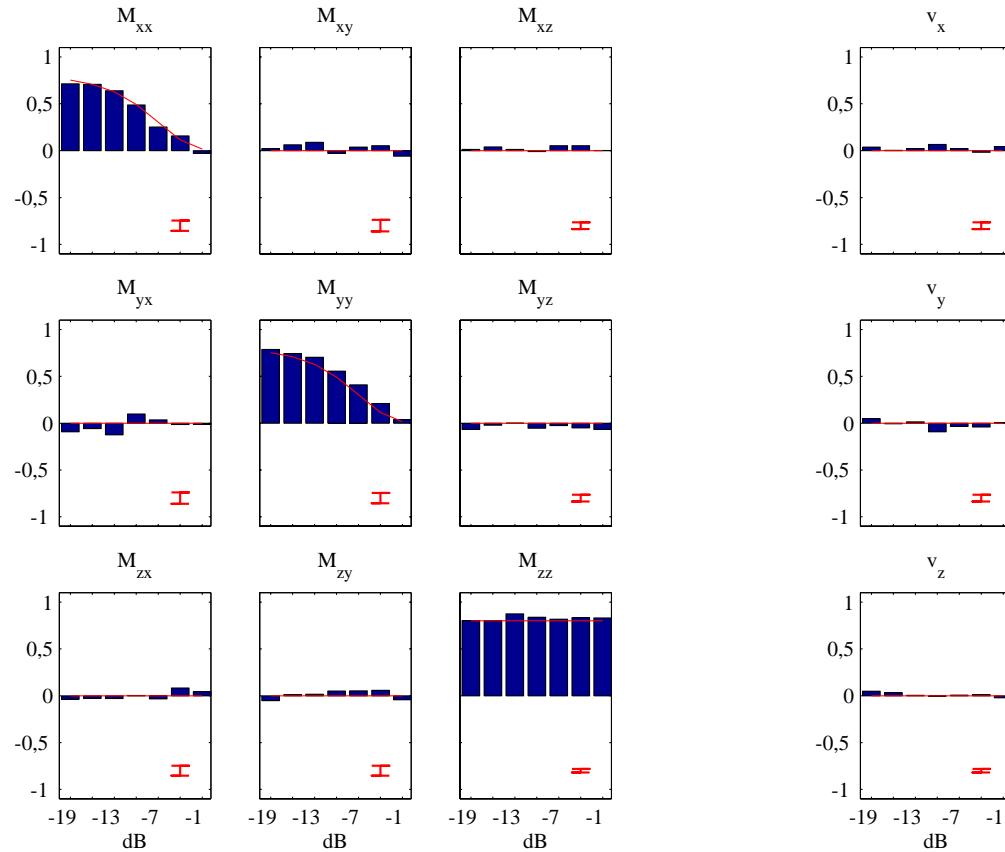


A. Fujiwara, P. Algoet, PRA **59**, 3290 (1999):

$$\vec{\sigma}' = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix} \cdot \vec{\sigma} + \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

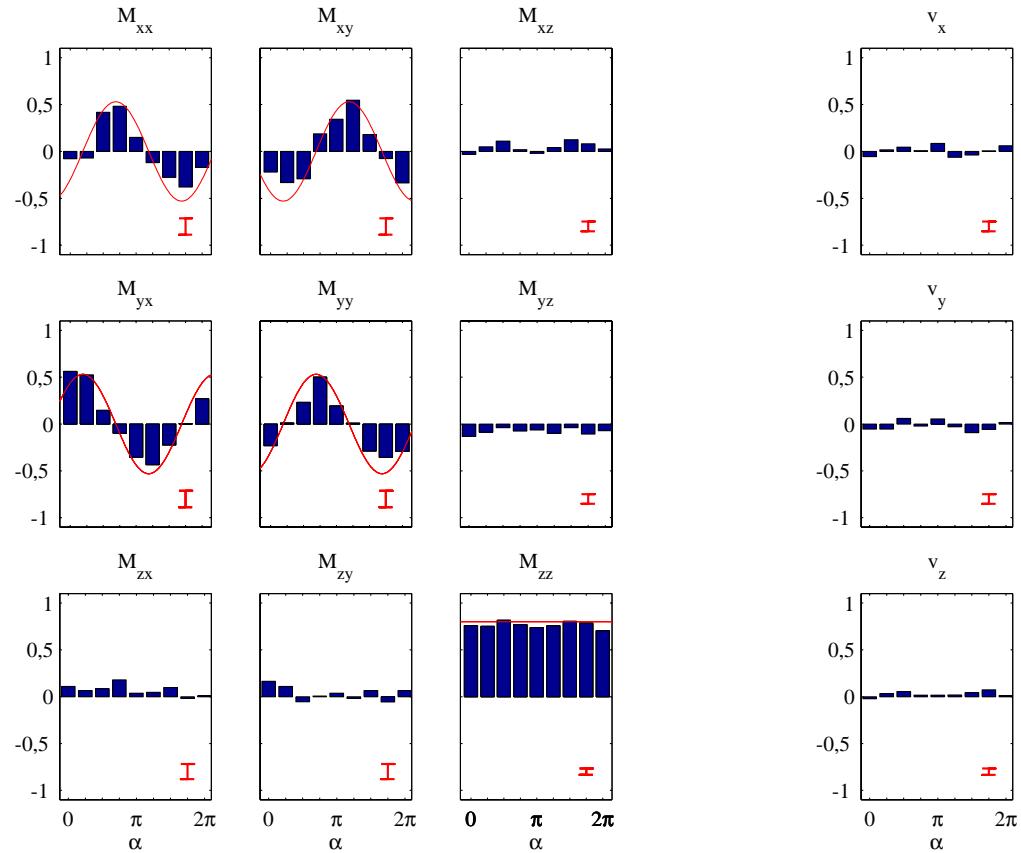
Realization of quantum channels

Variable phase damping



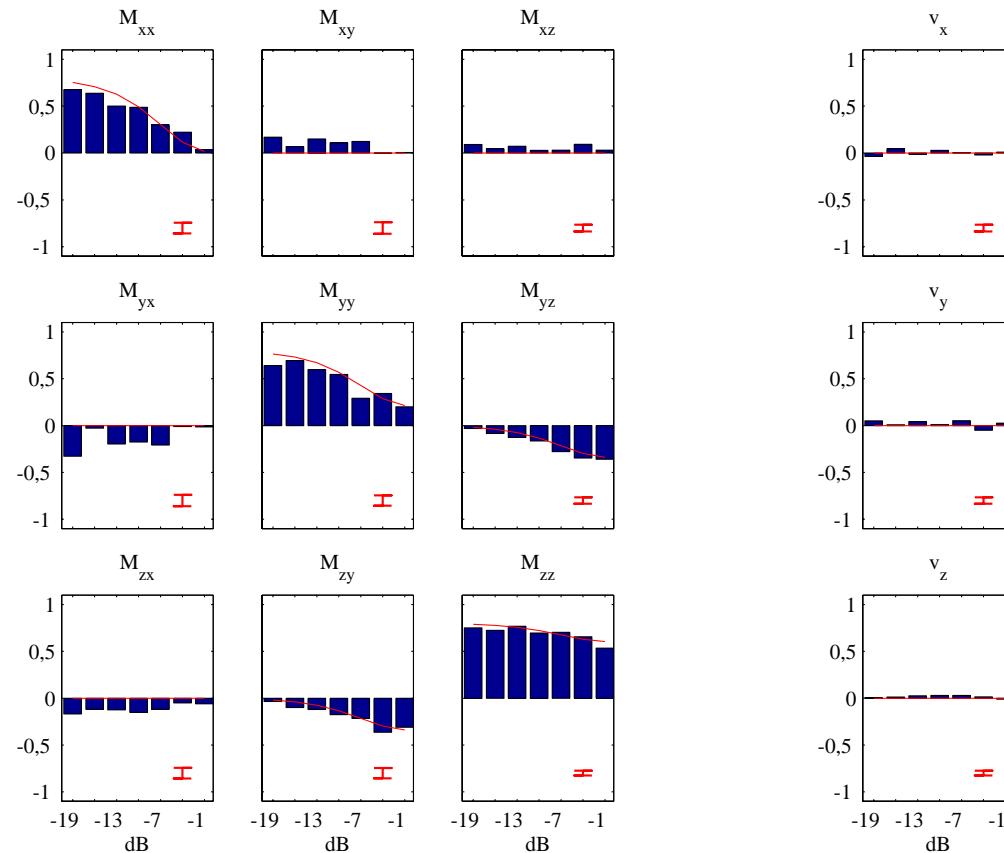
Realization of quantum channels

Fixed phase damping plus arbitrary polarization rotation



Realization of quantum channels

Phase damping in arbitrary plane. Here $\theta=\pi/6$



W. Neuhauser

P.E. Toschek

Ch. Wunderlich

D. Reiß

K. Abich

Ch. Balzer

A. Braun

Th. Hannemann

A. Keil

S. Thöming

Ph. Leick

F. Mintert

M. Riebe

F. Scharnberg



Overview

- New concepts
 - Conditional dynamics in ion traps using microwaves.
 - Spin resonance in ion traps: designed molecules.
 - Simultaneous sideband cooling of all vibrational modes.
- Experiments with Yb^+
 - Self-learning estimation of quantum states.
 - Realization of quantum channels.
 - Quantum Zeno paradox.
- Experiments with Ba^+
 - Efficient, robust Raman cooling.
 - Coherent excitation of E2 transition.